1

THEORY OF RANDOM SOLID STATES

CEDEX

Abstract

This text is a non-technical, elementary introduction to the theory of glassy phases and their ubiquity. The aim is to provide a guide, and some kind of coherent view, to the various topics which have been explored in recent years in this very diverse field, ranging from spin or structural glasses to protein folding, combinatorial optimization, neural networks, error correcting codes and game theory.

1.1 A few landmarks

Structural glasses

From rubber to spin glass and proteins

et al

N

$V\ r_i, r_j$

et al

CuMn

Mn

et al

et al

100

et al

Networks of interacting individuals: global equilibrium

N

i, j

J_{ij}

J_{ij} –

i

S_i

S_i – i j

S_i S_j

$-J_{ij}S_iS_j$

i, j, k $J_{ij}J_{jk}J_{ki}$

$_N$

N

N

$_N$

N

i

S_i S_i

S_i –

E $-\sum_{1 \le i \le j \le N} J_{ij}S_iS_j$

J_{ij}

et al

et al

E_0 N $-.$ $N^{3/2}$

$-N^2$

$-E/T$

E_1 E_0
N

et
al

et al

et al

N

N

N

N

Networks of interacting individuals: dynamics

global

et al

et al *et al*

et al

4

symmetric i j
j i

1.2 Tools and concepts

Statistical description

x $\rho\ x$ $\sum_i \langle \delta\ x - x_i\ \rangle$
x_i i

ρ x
x r
N

N

N N δN
$<< \delta N << N$

14

Physics without symmetry: equilibrium.

et al

N

$\pm$

T_c

$< S_i >$ $< . >$

T_c

$< S_i >/$

- $< S_i >$

q $/N$ $\sum_i < S_i >^2$

- α

  i $< S_i >_\alpha$

$q_{\alpha\alpha}$ $/N$ $\sum_i < S_i >^2_\alpha$ α

N
x_i N
N y_j x
y x y

x r_1
y r_1 r
x y

Replicas

et al

n Z^n

n

$n \rightarrow$

n

S_n n

n

$P\ q$

q

$P\ q$

et al *et al*

S_i q $/N\ \sum_{i=1}^{N} S_i S_i'$

$P\ q$

$P\ q$ $\delta\ q$

- $P\ q$ $\delta\ q - q_0$
  q_0
- $P\ q$
  $x\delta\ q - q_0$ $- x\ \delta\ q - q_1$
  q_1

q_0 q_0

- $P\ q$ $xp\ q$
  $- x\ \delta\ q - q_1$

Z^n

f

S_c f

m

F m m

S_c f S_c f

F m

Physics without symmetry: dynamics

α

$q \quad {}_{t\to\infty} \quad {}_{N\to\infty} < S_i \ t \ S_i \quad >$

$\alpha \quad t$

$q \quad {}_{N\to\infty} \quad /N \ \sum_i < S_i >_\alpha < S_i >_\alpha$
t

et
$al \quad t$

$t_w \quad t_w \quad \tau$
$C \ t_w \quad \tau, t_w \quad {}_{N\to\infty} \quad /N \ \sum_i < S_i \ t_w \ S_i \ t_w \quad \tau \ >$

$t_w \quad t_w$

t_w τ
$q \quad {}_{\tau\to\infty} \quad {}_{t_w\to\infty} C \ t_w \quad \tau, t_w$

${}_{\tau\to\infty} C \ t_w \quad \tau, t_w \quad t_w$

PVC

$t_w \quad \tau$

et al

P q

Simulations

et al

1.3 Directions

Physical glasses

t_w

ℓ t_w ℓ $\sqrt{t_w}$

t_w

t_w τ $\tau \ll t_w$ $\tau \gg t_w$

$\tau \ll t_w$

Random systems

et al *et al*

et al

et al

The unreasonable inefficiency of mathematics

ν / /

Consilience

Acknowledgments

References

Random Structures Algorithms **18**
Modelling brain function
Reference Frame *Physics Today*

Science **267**

Rev. Mod. Phys. **66**
J. Phys. (France) **2**

J. Physique **4**
Theory of Financial Risks

Heidelberg Colloquium on Glassy Dynamics and Optimization

Physica **A 276**
Phys. Rev. Lett. **84**
Phys. Rev. Lett. **75**
Phys. Rev. Lett. **71**
J. Phys. **27**
Phil. Trans. R. Soc. London **280 A**

Metastable liquids

Science **297**
J. Phys. **5**
Spin Glasses

J. Phys. **20**
Phys. Rev. **38**
Europhys. Lett. **26**

Physica **210**

Phys. Rev. Lett. **81**

Adv. Phys. **45**

Commun. Math. Phys. **230**

Comm. Math. Phys. **233**

Phys. Rev. Lett. **88**

Proc. Nat. Acad. Sci. USA **79**

Phys. Rev. **B17**

Science **220**

Phys. Rev. Lett. **78**

Introduction to the Theory of Neural Networks

Phys. Rev. Lett. **85**

J. Phys. **A27**

J. Phys. **A27**

J. Phys. **17**

First Steps in Glass Theory *More is Different*

Science **301**

J. Phys. Lett. **46**

Phys. Rev. Lett. **82**

Phys. Rev. Lett. **52**

J. Physique **45**

Europhys. Lett. **1**

Spin Glass Theory and Beyond

Statistical Physics of Spin Glasses and Information Processing

Phys. Rev. Lett. **85**

Phys. Rev. Lett. **43**

J. Phys. **13** *ibid* *ibid*

Phys. Rev. Lett. **79**

Contribution to this volume

Phys. Rev. Lett. **35**

The Generalized Parisi Formula *Compte Rendus de l'Academie des Sciences*

Spin Glasses: A Challenge to Mathematicians

Spin Glasses and Random Fields