Supplemental material for 'Blast dynamics in a disipative gas'

Henderson constitutive relation

The dense transport framework for inelastic hard spheres suggests the constitutive relation

$$p = n\Theta Z(n) \tag{1}$$

where $Z(n)$ stems from the finite compressibility of the gas. In the dilute limit, $Z(n \to 0) \to 1$ and the relation above becomes the ideal gas law (note that k_B does not appear since Θ is an energy rather than a thermodynamic temperature). However, $Z(n)$ diverges at finite n to account for the increase of pressure due to steric effect; while there is no known expression for $Z(n)$ that may apply in all conditions, we choose the classical Henderson relation [35]

$$Z(n) = \frac{1+\phi(n)^2/8}{(1-\phi(n))^2} \tag{2}$$

with the volume fraction $\phi(n) = n\,\pi\,\sigma^2$. This relation is found to provide sufficient agreement between theoretical and numerical results, despite being derived under assumptions (such as local equilibrium) not expected to hold in our system. Other valid candidates, some incorporating a singularity at close packing, can be found in [36] and yield comparable results.

Extended Rankine-Hugoniot conditions

Assuming the system to be quasi-1d near the interface, it is possible to integrate Eqs.(1) between coordinates r_1 and r_2 such that $\dot{r}_1 = \dot{r}_2 = \dot{R}$, and obtain flux difference equations:

$$\begin{aligned}
&\Big[n(u-\dot{R}) \Big]_{r_1}^{r_2} = 0\\
&\Big[n(u-\dot{R})\,u + P \Big]_{r_1}^{r_2} = 0\\
&\left[n(u-\dot{R})\left(\frac{u^2}{2}+\frac{2}{d}\Theta\right) + u\,P \right]_{r_1}^{r_2} = -\int_{r_1}^{r_2} dr\Lambda(r)
\end{aligned} \tag{3}$$

Letting $r_1 = R(t)-\epsilon$ and $r_2 = R(t)+\epsilon$ with $\epsilon \to 0$ gives a system with one non-trivial solution corresponding to finite compression at the interface, the usual Rankine-Hugoniot jump conditions [32] – in fact, the singular layer thus delimited is the shock front, of microscopic width, where hydrodynamical models fail due to the mixing of particles in different states. But the equations above hold for any $r_1 = R(t) - x$ with finite x, provided that distance is small enough compared to the radius of curvature of the interface that local unidimensionality can be maintained, and assuming higher order hydrodynamical terms play no significant role in that region. We thus extend this description to characterize not only the jump between the boundaries of the shock front, but also the continuous variation of the fields through the cooling region, up to the point of maximal compression.

Intermediate scaling regimes

As discussed in the main text, the similarity regimes obeyed by the blast, conservative or dissipative, are driven by central pressure and orthoradial momentum exchanges; if both vanish, a new conservation law appears and controls the scaling regime: conservation of radial momentum per angular sector (or solid angle in spatial dimension $d = 3$). In addition, for $\alpha < 1$, if the central pressure is eliminated before orthoradial exchanges, one should expect an exponent $\delta < 1/(d+1)$. This may provide an explanation for the slower growth observed experimentally [13, 14]: in that case, the blast is created with an initial central hole, which nullifies the central pressure, and it furthermore never reaches the MCS fixed point due to the non-zero external temperature, which eventually causes the shock to decay. In other settings, orthoradial exchanges in the shell usually vanish *before* the central pressure caused by non-accreted energetic particles, and an intermediate regime known as the Pressure-Driven Snowplow (PDS) arises with $\delta = 2/(d\gamma+2) \geq 1/(d+1)$ [8]. It is self-similar of the second kind, varying with the adiabatic index γ. Fig. S1 provides a numerical validation of this succession of regimes in a granular gas with a standard $\gamma = 1+2/d$, so that $\delta = 2/(d+4) = 2/7$ in dimension three.

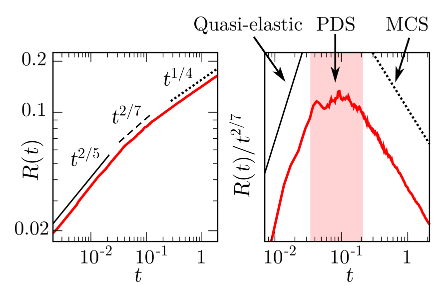

FIG. S1: Scaling of the radius $R(t)$ in a three dimensional simulation ($d = 3$). There are three successive regimes: quasi-elastic $t^{2/(d+2)}$ (solid line) before dissipation becomes significant, then Pressure-Driven Snowplow (PDS, dashes) $t^{2/(d\gamma+2)}$, and Momentum-Conserving Snowplow (MCS, dots) $t^{1/(d+1)}$. Left: Raw data. Right: Data rescaled by the PDS law, to evidence that regime over one decade (shaded region).

Linear stability analysis

In addition to the perturbation of the hydrodynamical fields (8), we must consider the perturbed radius

$$R(\theta,t) = R(t)\,(1 + \delta\mathrm{R}\,\cos(k\theta)\,t^s) \tag{4}$$

As the normal to the interface is now different from $\mathbf{e}_r$, the orthoradial velocity field can be non-zero (as seen in Fig. 1, particle velocities tend to stay aligned with the normal), and we must consider a pertubation $\delta\mathrm{V}_\perp$. The coupling between the four perturbations δM, $\delta\mathrm{P}$, $\delta\mathrm{V}_\mathrm{r}$ and $\delta\mathrm{V}_\perp$ is expressed as a matrix whose coefficients are functions of λ. In the incompressible limit, $M(\lambda) = M_{\mathrm{rcp}}$ and there is no density perturbation $\delta\mathrm{M} = 0$. The unperturbed profiles are then given by Eqs. (7). The matrix equation for the remaining perturbations then takes the form (omitting the dependence in λ)

$$\begin{pmatrix} 1 & 0 & 0 \\ V-\delta & 0 & M^{-1} \\ 0 & V-\delta & 0 \end{pmatrix} \frac{d}{d\ln\lambda} \begin{pmatrix} \delta\mathrm{V}_\mathrm{r} \\ \delta\mathrm{V}_\perp \\ \delta\mathrm{P} \end{pmatrix} = -\begin{pmatrix} d & -k^2 & 0 \\ s-1+2V+V' & 0 & (d+1)M^{-1} \\ 0 & s-1+V & M^{-1} \end{pmatrix} \begin{pmatrix} \delta\mathrm{V}_\mathrm{r} \\ \delta\mathrm{V}_\perp \\ \delta\mathrm{P} \end{pmatrix} \tag{5}$$

where we define $\Psi' = d\Psi/d\ln\lambda$ for any field $\Psi(\lambda)$.

Rankine-Hugoniot conditions on the perturbed interface allow to specify boundary values for the perturbations

$$\begin{aligned} \delta\mathrm{V}_\mathrm{r}(1) &= \frac{s}{\delta}V(1) - V'(1), \quad \delta\mathrm{V}_\perp(1) = -V(1) \\ \delta\mathrm{P}(1) &= \frac{2s}{\delta}P(1) - P'(1) \end{aligned} \tag{6}$$

and we must solve the equations above numerically by shooting from that interface toward the internal boundary with the central cavity, optimizing the parameter s in the matrix to satisfy the condition

$$\lim_{\lambda\to\lambda_i} P(\lambda)\,\delta\mathrm{P}(\lambda) = d\delta^2(1-M)(1-M^{-1}) \tag{7}$$

The above condition guarantees null pressure (including the perturbation) at the interface with the cavity. The location of the boundary $\lambda_i = R_i/R$ can be computed from the hydrodynamic profiles using the condition $u(R_i,t) = \lambda_i\dot{R}$ since, at the boundary with an empty region, the phase velocity of the wave must be equal to the flow velocity.