

# Fast equilibrium switch of a micro mechanical oscillator

## Supplementary Informations

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### ENERGETICS MEASUREMENTS

For a system in contact with a heat bath described by a classical Langevin equation, stochastic energetics (Ref.14 of the main text) provides the framework to quantify work and dissipation. Along these lines, we calculate the thermodynamics quantities from measurements of the position of the sphere in the course of the transformation for a given protocol. The sphere attached to the cantilever has a total energy:

$$E = U(x, F) + K(v) = \frac{1}{2}\kappa x^2 - Fx + \frac{1}{2}mv^2 \quad (\text{S.1})$$

where  $U(x, F)$  is the potential energy,  $K(v)$  the kinetic energy and  $F$  the electrostatic force applied to the system.

The elementary work done on the system reads

$$dW = \frac{\partial E}{\partial F}dF = -xdF, \quad (\text{S.2})$$

and yields the following integrated expression over the duration of the protocol

$$W(t_f) = \int_0^{t_f} \frac{\partial E}{\partial F} \circ \frac{\partial F}{\partial t} dt \quad (\text{S.3})$$

where  $\circ$  denotes Stratonovich integral.

Similarly, the two contributions to the heat are given by:

$$dQ_x = \frac{\partial E}{\partial x}dx = \kappa x dx - F dx \quad (\text{S.4})$$

and

$$dQ_v = \frac{\partial E}{\partial v}dv = mv dv. \quad (\text{S.5})$$

We thus get, for the whole protocol duration

$$Q(t_f) = \int_0^{t_f} \frac{\partial E}{\partial x} \circ \frac{\partial x}{\partial t} dt + \int_0^{t_f} \frac{\partial E}{\partial v} \circ \frac{\partial v}{\partial t} dt. \quad (\text{S.6})$$

It should be emphasized that the data reported in Fig.4 are the averages of these fluctuating quantities, taken over 5000 realizations of the experiment.

### ESE PROTOCOL

As a first approximation, the system made of the AFM cantilever plus the sphere can be modeled by a simple harmonic oscillator of resonant frequency  $f_0 = (\kappa/m)^{1/2}/(2\pi) = 2750$  Hz. The validity of our model breaks down for frequencies exceeding  $f_0$ .

We monitor in real time the evolution of the system during the relaxation towards the new equilibrium state, and get therefore the expression for the probability density distribution  $P(x, v, t)$ . Its dynamics obeys the Kramers equation associated to the Langevin equation Eq.(1) in the main text :

$$m[\partial_t + v\partial_x - \omega_o^2(x - x_0)\partial_v]P = \gamma\partial_v[vP + k_B T\partial_v P] \quad (\text{S.7})$$

where  $\omega_o^2 = \kappa/m$  and  $x_0(t) = F(t)/\kappa$ . To work out the appropriate function  $F(t)$ , we proceed as follows: we find out a specific exact solution  $P_e(x, v, t)$  of (7) that fulfills our boundary conditions  $x_0(t_i) = 0$  and  $x_0(t_f) = x_f$ , and infer from this solution the external force that shall be applied.

For the sake of simplicity, we look for a solution of the form

$$P_e(x, v, t) = \exp[-\alpha(x, t) - \beta v^2 - \delta(t)v], \quad (\text{S.8})$$

with  $\beta = m/(2k_B T)$  constant. Combining Eqs. (7) and (8), we deduce that the function  $\alpha(x, t)$  should be of the form  $\alpha(x, t) = \alpha_0(t) + \beta\omega_o^2 x^2 + \Delta(t)x$  with  $\Delta(t) = -\dot{\delta} - \gamma\delta/m - 2\beta\omega_o^2 x_0$ . The term  $\alpha_0(t)$  accounts for the normalization. Noticing that  $\dot{\Delta} = \omega_o^2 \delta$ , we have a direct relation between  $x_0(t)$  and  $\Delta$ :

$$x_0(t) = -\frac{1}{2\beta\omega_o^2} \left( \Delta + \frac{\gamma}{m\omega_o^2} \dot{\Delta} + \frac{1}{\omega_o^2} \ddot{\Delta} \right). \quad (\text{S.9})$$

For consistency, we set the boundary conditions of the  $\Delta$  function as:  $\Delta(0) = 0$  and  $\Delta(t_f) = -2\beta\omega_o^2 x_f$ ,  $\dot{\Delta}(0) = \dot{\Delta}(t_f) = 0$  and  $\ddot{\Delta}(0) = \ddot{\Delta}(t_f) = 0$ . To fulfill those six constraints, we choose a polynomial of order five:

$$\Delta(t) = 2\beta\omega_o^2 x_f s^3 (-10 + 15s - 6s^2), \quad (\text{S.10})$$

with the dimensionless time  $s = t/t_f$ , varying from 0 to 1. From Eq. (9), we infer  $x_0(t)$ , and thus the expression  $F(t)$  of Eq.(2) of the main text. Note that the

ESE protocol is worked out here in its simplest setting in which a harmonic potential has been used and the dynamics is controlled by only one parameter, i.e.  $F(t)$  in this specific example. This is the reason why the results can be readily recovered directly from the Langevin equation, averaging over noise realizations to work with the mean position  $\langle x \rangle$ . Imposing the desired evolution for this quantity leads directly to Eq. (9), which can be supplemented with the polynomial choice (10) [1, 2].

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