Paris-Sud / Paris-Saclay University
M1 General Physics

Year 2017-2018
STATISTICAL PHYSICS
Partial exam 2018

2 HOUR 15 MINUTES
DOCUMENTS, POCKET CALCULATORS AND ANY ELECTRONIC DEVICE NOT ALLOWED
Concise but explicative answers expected throughout. No bonus for verboseness

1 Basic questions

1) What does the notion of *universality* mean in statistical physics ? Provide some examples

2) In a magnetic system, how is c_B, the specific heat at fixed magnetic field, related to the free energy F and the temperature T ?

3) What are the key steps in Rudolf Peierls' exact treatment of the two dimensional Ising model?

4) Define the 6 critical exponents α, β, γ, δ, ν and η for a fluid.

5) We consider a Gaussian random variable X with mean 1 and standard deviation 2. Compute $\langle e^{2X}\rangle$, $\langle X^2\rangle$, $\langle X^3\rangle$ and $\langle X^4\rangle$.

6) For an arbitrary system, show that the specific heat at constant volume, c_v, is related to the fluctuations of a quantity to be specified. Conclude on the sign of c_v. Then, invoking extensivity arguments, quantify the relative importance of fluctuations, in systems of increasing size. What can we conclude from such an argument?

7) *(bonus question, in case you are finished before the end)* Compute

$$\int_{-\infty}^{\infty} \frac{1}{1+x^4}\, dx \tag{1}$$

◇

2 Gibbs-Bogoliubov inequality

The Gibbs-Bogoliubov inequality reads

$$F \leqslant F_0 + \langle H - H_0\rangle_0 \tag{2}$$

1) What is the meaning of the different symbols ?

2) What should H and H_0 have in common ?

3) What does happen when $H_0 = 0$?

4) Show the above $H_0 = 0$ relation by a direct calculation that thus does not make use of the general statement (2). Introduce $\mathcal{N}$, the total number of microscopic states in this problem.

◇

3 Landau theory

We consider a Landau free energy of the form

$$\mathcal{R} = \frac{1}{2}\, a_2\, \phi^2 - \frac{1}{3}\, a_3\, \phi^3 + \dots \tag{3}$$

where ϕ is the order parameter. The coefficient a_2 is temperature dependent, of the form $a_2 = \widetilde{a}_2\,(T - T_c)$; a_3 is constant and positive: $a_3 > 0$.

1) What is generically the next term in the expansion? Is it necessary to include it in the analysis ?

2) Do you know a system where such a treatment would be relevant ?

3) Discuss the scenario encoded in (3), as temperature changes. Provide a graph of the relevant free energy profiles in the plane $\phi, \mathcal{R}$, at well chosen temperatures.

4) Compute explicitely the characteristic temperature (or temperatures) appearing in this formulation.

◇

4 Legendre transforms

1) Take a given eligible function. What is the Legendre transform of its Legendre transform?

2) Compute the Legendre transform of the function $f(x) = e^{x-1}$. How are the curvatures of these two functions related ?

3) Sketch graphically the Legendre transform of function A in the graph below. What are the main features of both functions ?

4) Attempt the same graphical construct for function B. What is the strange phenomenon taking place ? What is it ascribable to?

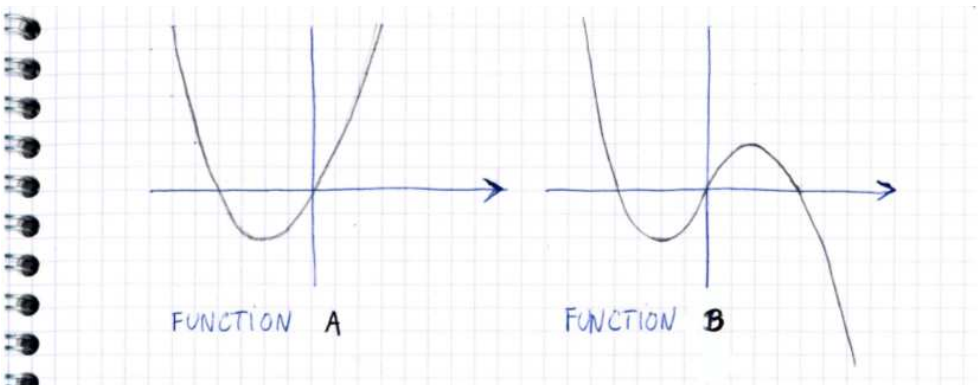

Figure 1: Adrien Marie Legendre... transformed