M1 General Physics
Year 2019-2020
STATISTICAL PHYSICS
Partial exam 2020

2 HOURS 15 MINUTES
DOCUMENTS, POCKET CALCULATORS AND ANY ELECTRONIC DEVICE NOT ALLOWED
Concise but explicative answers expected throughout. No bonus for verboseness

1 Basic questions

1) Statistical physics treatments of phase transitions do not go without problems. What are the main ones?

2) Why is Ising model of particular interest in Physics?

3) What are liquid crystals? For studying the isotropic-nematic transition of liquid crystals *confined in a planar membrane*, what would the order parameter be?

4) What could we call the ideal magnet equation of state, in a magnetic field B at temperature T? Start from writing explicitly the Hamiltonian and then derive such a relation, in a canonical ensemble calculation.

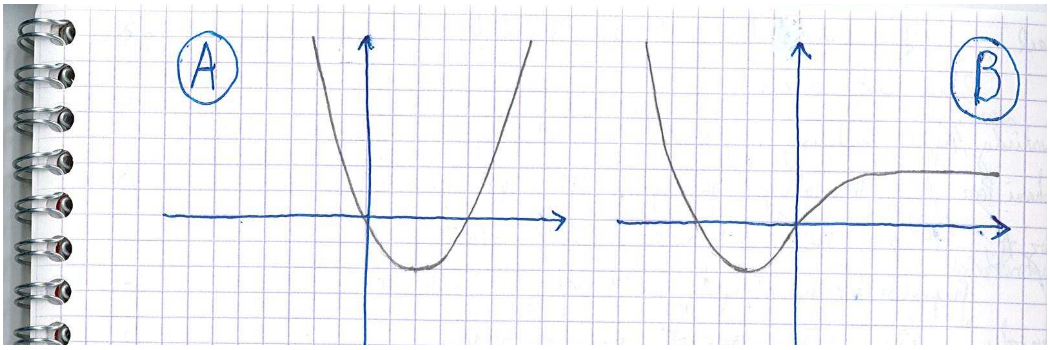
5) We consider an arbitrary system of interacting spins in an external magnetic field, at a fixed temperature T. Relate the mean magnetization to the free energy. What are the "natural" variables for the free energy?

6) In a magnetic system, how is c_B, the specific heat at fixed magnetic field, related to the free energy F and the temperature T ?

7) Same question as above, for c_M, the specific heat at fixed magnetization. What is here the relevant thermodynamic potential? How is it related to the free energy of the previous question?

8) Sketch graphically the Legendre transform of function A in the graph below.

9) Same question for function B. Do you obtain the graph of a function? Why?



10) Compute

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} \tag{1}$$

by two methods, one of them using the residue theorem.

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2 Gaussian calculus

We consider a Gaussian random variable x with mean m and standard deviation σ. From this probability density function, we define the average denoted below by brackets.

1) Compute $\langle e^x \rangle$. Compare to $e^{\langle x \rangle}$. Does the corresponding inequality depend on the Gaussian statistics? When do we have equality? Explain.

2) Compute $\langle e^{-x} \rangle$.

3) In the remainder, we set $m = 0$. Relate $\langle x f(x) \rangle$ to $\left\langle \frac{\partial f}{\partial x} \right\rangle$, where f is an "arbitrary" function.

4) Use the above result to compute $\langle x^4 \rangle$, and $\langle x^6 \rangle$.

5) What is $\langle x^n \rangle$ for an integer n?

6) Recover then the result of the first question for $\langle e^x \rangle$ by a Taylor expansion argument.

⋄

3 Magnetic domain walls

In a magnetic system, the local mean magnetization m can be a function of a Cartesian coordinate x. The boundary conditions are such that $m \to -1$ for $x \to -\infty$ while $m \to 1$ for $x \to \infty$. How can such boundary conditions be achieved experimentally?

We do not assume any particular form for the underlying Hamiltonian, but we instead write the total free energy $\mathcal{F}$ of the system as a functional of $m(x)$, such that

$$\mathcal{F} = \int \left\{ -\frac{1}{2} m^2 + \frac{1}{4} m^4 + \frac{1}{2} \left(\frac{dm}{dx} \right)^2 \right\} dx. \tag{2}$$

How can one proceed to find the optimal profile $m^*(x)$, compatible with the boundary conditions? Write $m^*(x)$ explicitly.