Should be written on a separate paper.
Concise but explicative answers expected throughout. No bonus for verboseness

1 Basic questions

1) What does *universality* mean in the statistical physics treatment of phase transitions?

2) Discuss competing effects that may be at work in phase transitions. Propose two different scenarios where in each case, the mechanisms would differ (different competing effects in the two situations).

3) Are there phase transitions for which temperature plays no role and if so, which ones?

4) Sketch graphically the Legendre transform of function A in the graph below. Comments?

5) Take $f(x) = -1/x$ for $x > 0$. Compute analytically the corresponding Legendre transform $\widehat{f}(p)$. Is this compatible with the answer to the previous question?

6) Sketch graphically the Legendre transform of function B in the graph below
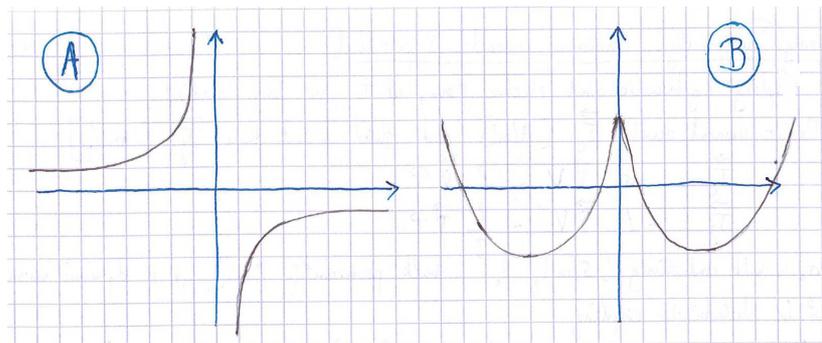
2 Partially miscible liquids - take 1

A two component mixture of species A and B may exist either in a fully mixed, or in a partially demixed state. Figure 1 displays how the thermodynamic potential $\mathcal{R}$ describing the experimental situation under study depends on the molar fraction of species B (varying between 0 and 1). Delineate the region of AB miscibility in a schematic temperature versus molar fraction diagram. Explain.
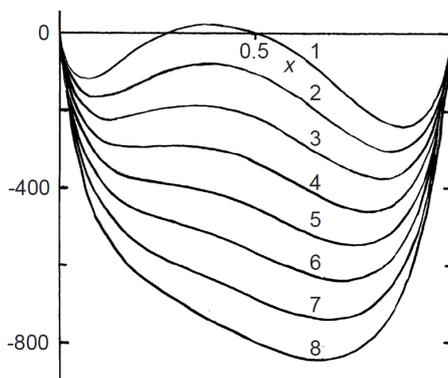

Figure 1: Plot of $\mathcal{R}$ as a function of x, the molar fraction of species B, for different temperatures: (1) 270 K; (2) 275 K; (3) 280 K; (4) 285 K; (5) 290 K; (6) 295 K; (7) 300 K; (8) 305 K. From S.R. Logan, *J. Chem. Educ.* **75**, 339 (1998).
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3 Partially miscible liquids - take 2

We are interested in mixing water and phenol (or methanol/decane etc.) at a given pressure. Quite a number of pairs of partially miscible liquids do lead to the same behaviour, shown in Fig. 2, in a temperature-concentration diagram (note that here, the concentration is dimensionless).
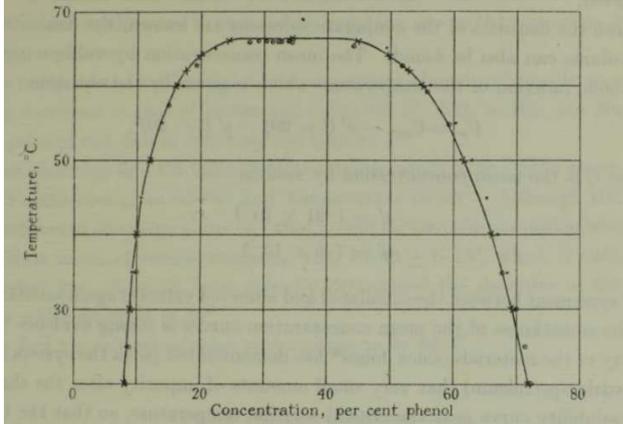

Figure 2: Water-phenol solubility curve at room pressure, from O.R. Howell, *Proc. R. Soc. Lond. A* **137**, 418 (1932). The x-axis is for the phenol concentration c, that ranges from 0 to 100%.

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1) The curve in Fig. 2 defines two regions: inside the "bell", and outside. One is for complete miscibility, the other for partial miscibility. On physical grounds, which is which ?

2) What is the thermodynamic potential suited to describe such a situation? Specify the variables.

3) Apply the Gibbs phase rule in each region. What do you conclude?

4 Partially miscible liquids - take 3

We seek for a simple framework that would rationalize the water/phenol miscibility measures. We write the thermodynamic potential as

$$\beta \mathcal{R} = \chi c(1-c) + c \log c + (1-c) \log(1-c). \tag{1}$$

where $1/\beta$ is the thermal energy kT.

1) What do the last two terms on the right hand side remind you of ? What can we then surmise concerning the origin of the other contribution in $\chi c(1-c)$?

2) For which χ value χ^* is $\mathcal{R}$ a convex-up function for all $0 < c < 1$?

3) Plot the typical $\mathcal{R}$-profiles for $\chi < \chi^*$ and $\chi > \chi^*$.

4) How does this relate to the measures reported in Fig. 2?

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5 Correlation function in an Ising chain

Take an open Ising one-dimensional chain (free boundary conditions), without a magnetic field, such that the couplings between nearest neighbors are all different. Starting from the direct calculation of the partition function, propose a method to compute the correlation function $\langle S_i S_j \rangle$. What is the correlation length here? Does this quantity signal a phase transition?