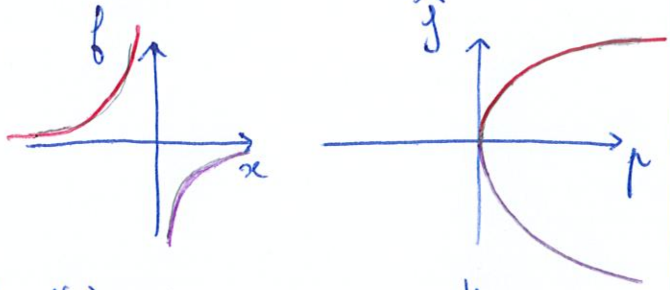
M1 GP - Statistical Physics - Partial exam March 2021

(1) 1) Energy-entropy or eg orientational vs positional entropy for I-N transition of hard rods

3) Yes, when only excluded volume interactions do matter

4)



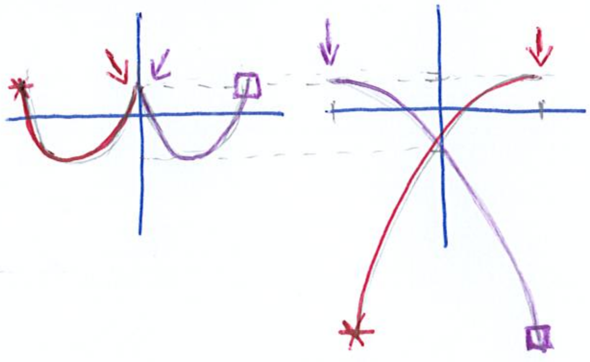
5) $f(x) = -1/x \;,\; x > 0 \;;\; p = \dfrac{df}{dx} = \dfrac{1}{x^2}$

$$x = \frac{1}{\sqrt{p}}$$

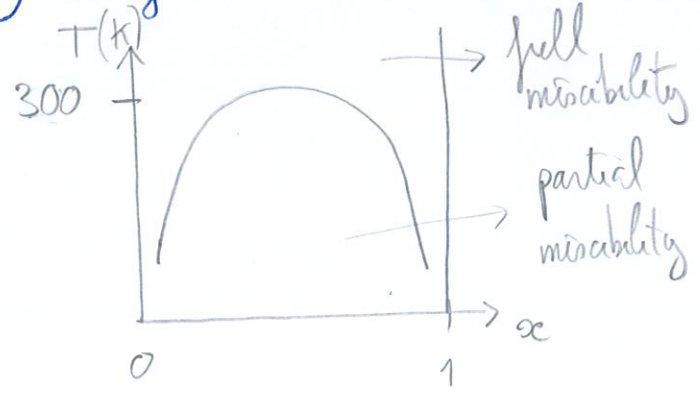
$$\hat{f}(p) = -\sqrt{p} - p \times \frac{1}{\sqrt{p}} = -2\sqrt{p}$$

this is compatible with the lower branch of the sketch above

6)



(2) Bitangent construction when $R(x)$ not convex up



(3) 1) Large T presumably favors mixing, thus same diagram as in question (2)

2) Gibbs free energy $G(T, P, c)$

$c \equiv$ phenol (molar) concentration

3) 2 species ; variance $= 2 + 2 - \varphi$,

φ number of phases.

$\varphi = 1$ (mixing) $\Rightarrow$ variance $= 3$,

T, P, c can be fixed

$\varphi = 2$ (demixing) $\Rightarrow$ variance $= 2$

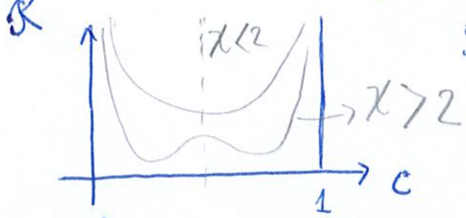
T, P chosen $\Rightarrow$ c fixed

(4) 1) Entropy of mixing ; enthalpy of interaction

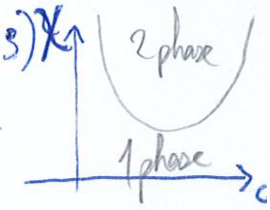
2) $\dfrac{\partial^2 \beta R}{\partial c^2} = -2\chi + \dfrac{1}{c} + \dfrac{1}{1-c}$ min for $c = 1/2$, value 4

$> 0 \;\; \forall c$ for $\chi < 2$; $\boxed{\chi^* = 2}$

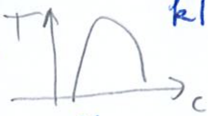
$> \text{or} < 0$ for $\chi > 2$;



3)



Note that $\chi = \dfrac{\text{energy}}{kT}$; $T \nearrow \Rightarrow \chi \searrow$

Hence  , same as Fig 2

(5) $\beta H = -\sum_{i=1}^{N} K_i S_i S_{i+1}$; $\tau_i \equiv S_i S_{i+1}$

$$Z = \sum_{S_1, \tau_1, \cdots \tau_{N-1}, \tau_N} e^{K_1 \tau_1} e^{K_2 \tau_2} \cdots e^{K_N \tau_N}$$

$$= 2 \prod_{k=1}^{N} 2\cosh(K_k) \quad ; \quad \text{take } j > i$$

$$\langle S_i S_j \rangle = \frac{1}{Z} \sum_{S_1 \cdots S_N} S_i S_{i+1} S_{i+1} S_{i+2} \cdots S_{j-1} S_j \, e^{-\beta H}$$

$$= \frac{1}{Z} \frac{\partial^{j-i} Z}{\partial K_i \, \partial K_{i+1} \cdots \partial K_{j-1}} = (\text{th} K_i)(\text{th} K_{i+1}) \cdots (\text{th} K_{j-1})$$

All K equal $\rightarrow \langle S_i S_j \rangle = (\text{th} K)^{j-i}$; $\xi = -\dfrac{1}{\ln \text{th} K}$