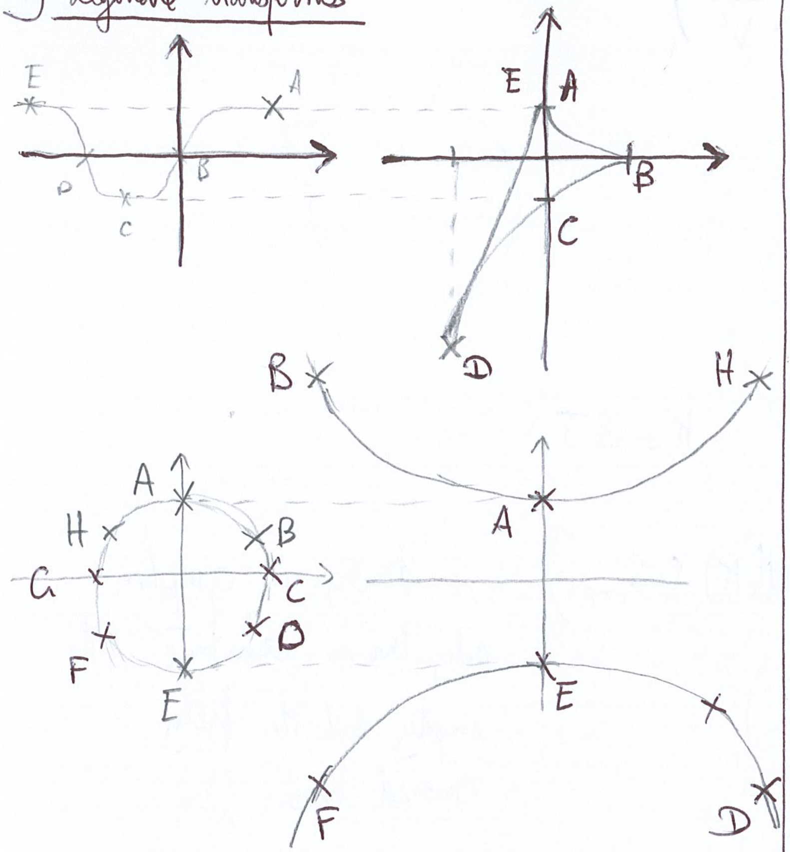
1°) Legendre transforms



2°) Phase transitions in hard sphere systems

1°) The transition is entropy driven, and does not result from energy-entropy competition.

2°) Let $n = \frac{N}{V}$: $\mu = f'(n)$

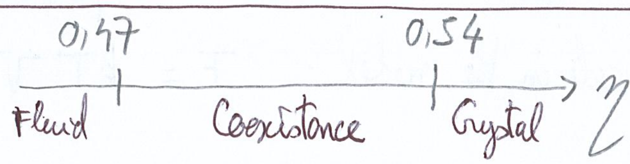
3°) $-P = f - n f'(n)$

$$\Rightarrow \mu = \frac{4}{3}\pi\sigma^3 \frac{df}{d\eta}$$

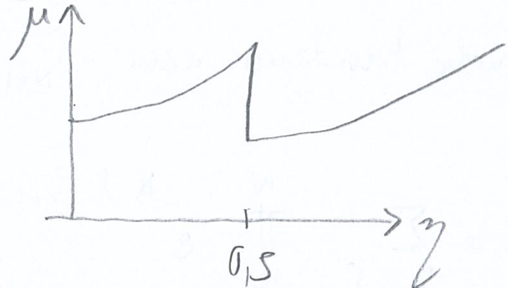
$$-P = f(\eta) - \eta \frac{df}{d\eta}$$

4°) The one with triangles is stable at low η, and is for the fluid branch. The other is for the crystal branch (stable at large η)

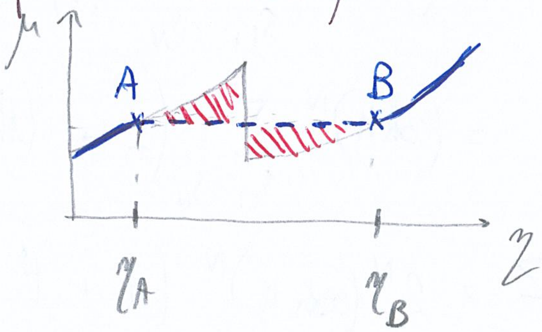
5°) Double-tangent. Hence the phase diagram

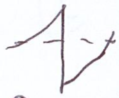


6°) If we restrict to stable branches:

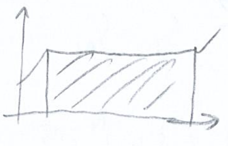


This does not account for demixtion → a plateau at constant μ.



We can go from A to B in 2 ways, either taking the plateau, or taking the 

$$\int_A^B \mu \, d\eta = \int_A^B \frac{df}{dn}\Big|_T d\eta = \frac{4}{3}\pi\sigma^3 \left(f(B) - f(A)\right)$$

Hence  = 

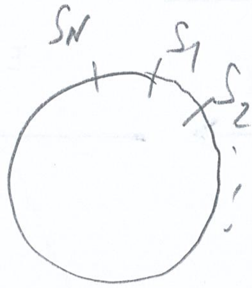
Thus like areas above  and 

→ reminiscent of Maxwell construction

7°) Samples 2 and 3 are fluid after 1 day. 4 shows phase coexistence, 5 also. 6 " crystallites.

After 4 days: 2 remains fluid, 3, 4 and 5 show fluid-crystal coexistence; 6, 7 and 8 → crystals; 9 and 10: coex glass + crystal

8°) the phase behavior does not depend on temperature, because there is no energy scale in the model. $F = kT\,V\, f\left(\frac{N}{V}\right)$



3 - Ising 1D - correlation functions

1°) Periodic boundaries mean $S_{N+1} = S_1$

2°) $$Z = \sum_{S_1 \cdots S_N} \prod_{i=1}^{N} e^{K S_i S_{i+1}} \qquad K = \beta J$$

$$= (\operatorname{ch} K)^N \sum_{S_1 \cdots S_N} \prod_{i=1}^{N} \left(1 + (\operatorname{th} K) S_i S_{i+1}\right)$$

; develop in graphs only two do contribute, the empty and the fully covered ones

$$= (\operatorname{ch} K)^N \sum_{S_1 \cdots S_N} \left(1 + (\operatorname{th} K)^N\right)$$

$$Z = 2^N (\operatorname{ch} K)^N \left(1 + (\operatorname{th} K)^N\right)$$

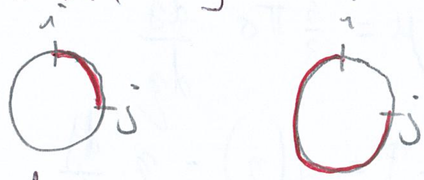
3°) $$\frac{F}{N} = -kT \ln Z \xrightarrow[N \to \infty]{} \underline{-kT \log(2 \operatorname{ch} K)}$$

same result as with free b.c.

4°) $$\langle S_i S_j \rangle = \frac{1}{Z} \sum_{S_1 \cdots S_N} S_i S_j \prod_{k=1}^{N} e^{K S_k S_{k+1}}$$

$$= \frac{1}{Z} (\operatorname{ch} K)^N \sum_{S_1 \cdots S_N} S_i S_j \prod_{k=1}^{N} \left[1 + S_k S_{k+1} (\operatorname{th} K)\right]$$

only 2 graphs contribute



$$= \frac{1}{Z} (\operatorname{ch} K)^N \left[(\operatorname{th} K)^{|j-i|} + (\operatorname{th} K)^{N-|j-i|}\right] 2^N$$

5°) $$\langle S_i S_j \rangle \xrightarrow[\substack{N \to \infty \\ i \text{ and } j \text{ fixed}}]{} (\operatorname{th} K)^{|j-i|}$$

hence a correlation length

$$\xi = -\frac{1}{\ln(\operatorname{th} K)}$$

a result we already knew.