PCS / Exam 2022

- Homogeneity of formulas : $\langle \tau^2 \rangle = \frac{a^3}{D^2}$ is not acceptable. where | $\tau \to$ time, $a \to$ length, $D \to$ diffusion coeff

- If X is gaussian $g(m, \sigma)$, you have to be able to write
$$\langle e^{\theta X} \rangle = e^{\theta m} e^{\theta^2 \sigma^2 / 2} \quad \text{for } \theta \text{ arbitrary}$$

- the Langevin equation may involve a variable $X(t)$, but the Fokker-Planck formulation "decouples" X and t, and it makes no sense to write
$$\partial_t p(\cancel{X}, t) = X(t)\, \partial_x p + \ldots$$

- You then have to be able to write the pdf, for $X \rightsquigarrow g(m, \sigma)$
$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-m)^2}{2\sigma^2}\right]$$

- If the x_i are iiD, then their variance obeys
$$V\left(\sum_{i=1}^{n} x_i\right) \overset{\text{iiD}}{=} \sum_{i=1}^{n} V(x_i) = n\, V(x) \quad \text{and not } n^2$$
But $\left\langle \left(\sum_{i=1}^{n} x_i\right)^2 \right\rangle \neq n \langle x^2 \rangle$

- Do not confuse the variance $V(x) \equiv \langle x^2 \rangle - \langle x \rangle^2$ with the 2^{nd} moment $\langle x^2 \rangle$

- For a gaussian distribution, the moments of order 4, 6 ... and possibly also 3 and 5 etc do not vanish ! Cumulant $\neq$ moment

- If you know the pdf of X, $p(X)$, you should be have to write the pdf of $S = e^X \Leftrightarrow X = \log S$, $\mathcal{P}(S)$
$$\mathcal{P}(S) = \frac{1}{S}\, p(\log S)$$
Same thing if $X = \log S + Cst$

- If you find that a variance vanishes, you better check your calculation. Apart for rather singular distribution, this should not happen

- Let $p(x, t \mid x_0, t_0)$ be the conditional distribution of X at time t, given $x(t_0) = x_0$. Let $f(x, t)$ be an arbitrary function

$$\langle f(X(t), t) \mid X(s), s \rangle = \int dx \; f(x,t) \, p(x, t \mid X(s), s)$$

no $\int dt'$ involved.

- To characterize a gaussian process $X(t)$, you should provide $\langle X(t) \rangle$, $\langle X(t) X(t') \rangle$.

Giving $\langle X(t) \rangle$ and $\langle X^2(t) \rangle$ is not enough

- For a multiplicative Langevin equation $\dot{x} = A(x) + \sqrt{2D}\, C(x) \xi(t)$ with $\langle \xi(t) \rangle = 0$; $\langle \xi(t) \xi(t') \rangle = \delta(t-t')$, the Stratonovich and Itō versions of Fokker-Planck equation are not the same:

<u>Strato</u>: $\partial_t p(x,t) = -\partial_x [A(x) p] + D \, \partial_x [C(x) \partial_x (C(x) p)]$

<u>Itō</u>: $\partial_t p(x,t) = -\partial_x [A(x) p] + D \, \partial_x^2 [C^2(x) p]$

- If the x_i are IID and exponentially, their sum is <u>not</u> exponentially distributed.

- It is not the best of ideas to use the word "easy" in your text, especially when the answer is wrong

- Pay attention to the difference between $\int_0^t f(t') dt'$ and $\int_0^t f(t) dt'$.

and... $\int_0^t f(t) dt$ is meaningless.