Stochastic processes
Outline

I - A probability bootstrap
1) Moments, cumulants, and their generating function
2) Dependent and independent variables
3) Sums of random variables and the central limit theorem
4) When the central limit theorem goes wrong (fat tails and extreme value statistics, Lévy flights, correlated variables)
5) Renormalization group view on the central limit theorem and its generalizations

II - Large deviations
1) Introductions
2) Revisiting the central limit theorem; where do "tails" start?
3) An entropy appears
4) Sanov's theorem
5) Correlated variables; Gärtner-Ellis theorem

III - The Langevin equation: physical aspects
1) Colloids and Brownian motion
2) The Langevin approach
3) Diffusion, relaxation and response
4) Dynamics of velocity relaxation at equilibrium
5) Conclusion; the generalized Langevin approach

IV - The Langevin equation: mathematical and computational aspects
1) Properties of the noise (the Langevin force)
2) Be wise... discretize!
3) Stochastic calculus (Itō, Stratonovich)
4) The tragic life and ignored legacy of Wolfgang Döblin
5) Numerical integration

V - Markov processes: master and Fokker-Planck equations
1) Definition
2) Markov processes
3) Markov chains (stochastic matrices, detailed balance, Monte Carlo simulations)
4) The Fokker-Planck equation (Kramers-Moyal expansion, forward/backward formulations, H-theorem)
5) Solving the Fokker-Planck equation (mapping to quantum mechanics)
6) Relation between the Langevin and Fokker-Planck equations: back to the Itō-Stratonovich dilemna

VI - Linear response theory intermezzo
1) Linear response and Onsager's intuition
2) Response functions and the fluctuation-dissipation theorem
3) Symmetry of the correlation functions (at equilibrium)
4) Back to the Langevin / Fokker-Planck description
5) Causality and Kramers-Kronig relations

VII - Functionals of stochastic processes: The Feynman-Kac correspondance
1) The Feynman-Kac correspondance (forward and backward formulations)
2) First-passage functionals

VIII - Introduction to stochastic thermodynamics
1) Historical perspective and context
2) The first law of stochastic thermodynamics
3) From Feynman-Kac to Jarzynski: the work fluctuation relation
4) Crooks relation
5) The stochastic entropy

IX - Introduction to martingales
1) Definition
2) Main properties of martingales
3) Application of the stopping theorem to first passage properties
4) Application to stochastic thermodynamics