Wireless Propagation in Buildings: A Statistical Scattering Approach

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Abstract— **A new approach to the modeling of wireless propagation in buildings is introduced. We treat the scattering by walls and local clutter probabilistically through either a relaxation-time approximation in a Boltzmann equation or by using a diffusion equation. The result is a range of models in which one can vary the trade-off between the complexity of the building description and the accuracy of the prediction. The two limits of this range are ray-tracing at the most accurate end and a simple decay law at the most simple. By comparing results for two of these new models with measurements, we conclude that a reasonably accurate description of propagation can be obtained with a relatively simple model. The most effective way to use the models is by combining them with a few measurements through a sampling technique.**

Keywords—**wireless, propagation, indoor, statistical model**

I. Introduction

Indoor wireless communication has been a subject of intense investigation in recent years in the context of both voice and data communication. One important problem in this area is the propagation of the wireless signal from the transmitter to the mobile receiver through the complicated indoor environment (for reviews see [1], [2], [3], [4]). Scattering from both walls and local clutter produce a heavily multipath channel in which the path-loss is very different from that in outdoor contexts. The characteristics of the propagation must be taken into account in determining the number of transmitters needed, the cell size, and the placement of the transmitters. In order to guide design and installation of indoor wireless systems, it is therefore important to develop effective models of wireless propagation inside buildings.

Several characteristics of the propagation are important for indoor wireless communication. The three most studied are the mean power— the signal intensity averaged in space over a distance of several wavelengths— the time delay spread— the spread in transit time from the transmitter to the mobile— and the fast fading— the fluctuations of the power about the mean caused by interference. For the purposes of this paper, we will discuss only the mean power, though some of our results can be extended to these other quantities. Our goal, then, is accurate coverage prediction for indoor propagation. In order to gauge whether a propagation model is sufficiently accurate, we take as a target that the error statistics— the difference between the mean power predicted by the model and the actual power— should have a mean less than 3 dB and a standard deviation less than 6 dB. These criteria are similar to those used previously [2], [3], [4].

Previous work modeling wireless propagation in buildings has emphasized two very different approaches for the mean power. First, a simple power-law decay of the path-loss has been considered where the exponent used is determined empirically from measurements. This approach is a natural extension of the standard power-law decay used in outdoor propagation: in the outdoor environment, the decay is $1/r^2$ up to a break-point and $1/r^4$ beyond that point where the break-point is determined by the interference of the direct and ground bounce rays. Several studies have investigated the utility of an empirical power-law approach indoors and have attempted to determine the appropriate power law for buildings of various types [4], [5], [6].

The second main approach to indoor propagation used previously is ray-tracing. In this approach one assumes, first, that the geometrical optics limit to the electromagnetic wave propagation is adequate. This will be true if the important objects inhibiting propagation are larger than a wavelength, which is certainly true of the walls and ceiling of a building. Diffraction effects can be included as a further refinement. For ray-tracing one further assumes that the location and reflection properties of all the important objects inhibiting propagation are accurately known. Then one finds all the possible paths connecting the transmitter and the mobile through the specified complicated environment. Ray-tracing models of propagation have been intensively pursued in recent years with considerable success [6], [7], [8], [9], [10], [11], [12], [13], [14].

Each of these two approaches makes a very different trade-off of *accuracy* versus *complexity*. The power-law model is extremely simple— no building information is used other than the general type used in choosing the exponent— but the prediction is insufficiently accurate according to our target criterion above. On the other hand, ray-tracing is considerably more accurate than the power-law model, but requires a great deal of specific information about the building— the locations of all the walls at a minimum and possibly the locations of other large objects, especially metal ones. Thus the trade-off that each of these approaches makes is extreme.

In this paper we introduce a range of models which make a trade-off of accuracy versus complexity which is intermediate between that of power-law decay or ray-tracing. We

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include partial information about the building— its shape and the typical wall separation, for instance— and treat the unknown aspects using a probabilistic model for the scattering. The range of models that we introduce has as its two limits ray-tracing on the most accurate side and a simple decay law on the least complex side. All of the models address the mean power distribution: the geometrical optics limit is taken and so wave-interference effects are neglected from the outset. We show that certain models within our range are sufficiently accurate to be relevant for coverage prediction.

In Section II we present our statistical scattering approach. A range of models is introduced first for the artificial but instructive case of one-dimensional buildings, and then generalized to two and three dimensional buildings. In Section III we present results for two of the two-dimensional models and compare their predictions to measurements of single floor propagation in several buildings. The comparison suggests that our target accuracy criteria can be met with the statistical scattering approach. Finally, in Section IV we conclude.

II. Statistical Scattering Approach

We wish to model propagation in a building for which we have only partial information: the location of obstacles is not sufficiently well specified to use ray-tracing. All the information we have about the building should be used in the propagation model, but any features for which we do not have information will be treated as random. Thus it is necessary to introduce a probabilistic, i.e. statistical, model. One way to view the problem is to consider an ensemble of buildings consistent with the limited knowledge that we have and then try to predict the typical behavior through an average over the ensemble.

The physical quantity we wish to model is the local power density which itself is simply proportional to the local energy density. Because we are neglecting wave-interference effects, this quantity obeys a classical transport equation. The natural way to write this equation is in terms of the distribution of signal energy at position $\mathbf{r}$ traveling in direction $\mathbf{v}$, $f(\mathbf{r},\mathbf{v},t)$; the desired local energy density is obtained by integrating f over $\mathbf{v}$. The time evolution of any classical distribution function [15] is given by the Liouville equation in free space or the Boltzmann equation in the presence of scattering:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}} = I\left[f(\mathbf{r},\mathbf{v})\right] \; . \qquad (1)$$

The left-hand-side is the uninhibited free-space motion while the effect of scattering is given by the collision integral I— a functional of the distribution function f— on the right-hand-side. In a specific building, the collision integral is determined by the location of the walls and other obstacles such as local clutter. The boundary condition on $f(\mathbf{r},\mathbf{v},t)$ that accompanies the differential equation (1) is simply that zero energy density is incident on the building from the outside.

As an example, the deterministic collision integral for a single wall parallel to the y axis at x_0 with reflection coefficient $r(\theta)$ is

$$\begin{aligned} I\left[f(\mathbf{r},v_x,v_y)\right] = \; & -r(\theta)\;|v_x|\;\delta(x-x_0) \\ & \times\left[f(\mathbf{r},v_x,v_y)-f(\mathbf{r},-v_x,v_y)\right] \end{aligned} \qquad (2)$$

where θ is the angle between $\mathbf{v}$ and the x axis. In this case, a particle which traverses the point x_0 has its x velocity reversed with probability proportional to $r(\theta)$. With accurate information about all of the obstacles in a building, a full deterministic collision integral can be specified. Then the Boltzmann equation description is exactly equivalent to ray-tracing; in fact, one of the standard ways of solving Boltzmann equations is through ray-tracing, more generally referred to as the method of characteristics [16].

In our situation, however, we do *not* know the exact location and properties of each obstacle. In this case, treatment using a deterministic collision integral or equivalently exact ray-tracing is not possible, and so we make a statistical approximation to the collision integral. Suppose that the scattering is distributed uniformly and that the average distance traveled between scattering events is l, a quantity known as the mean-free-path, so that the rate of scattering per unit time is $(l/c)^{-1}$ where c is the speed of propagation. Suppose further that after the scattering event the radiation is ejected into a distribution $f_{\text{out}}(\mathbf{r},\mathbf{v})$ which can in general be a functional of f. Then the collision integral takes the form

$$I\left[f(\mathbf{r},\mathbf{v})\right] \approx -\frac{f(\mathbf{r},\mathbf{v})-f_{\text{out}}(\mathbf{r},\mathbf{v})}{l/c} \; . \qquad (3)$$

This is the relaxation-time approximation familiar in electron transport in solids [17], where f_{out} is the distribution to which the collisions relax f. The form of $f_{\text{out}}(\mathbf{r},\mathbf{v})$ can depend on the type of collision: it could be isotropic for scattering from local clutter, have the outgoing angle determined by the incoming angle for specular scattering, or be identically zero if the signal is completely absorbed in the collision. To make the connection with ray-tracing, this statistical approximation to the collision integral is related to using random walks of length l instead of the true rays in the building [18].

We emphasize that our introduction of a statistical treatment of scattering is connected to the lack of information about obstacles in the building. In this regard, the statistical model should be no less accurate than, for instance, a ray-tracing approach in which the unknown objects are simply left out. The main advantage of the statistical approach is that the main effect of the unknown objects is incorporated in an average way without requiring detailed knowledge of the objects. There are, however, two situations in which the applicability of a statistical approach is limited. First, in developing these models we assume that the walls are fairly transmissive: very reflective walls cannot be treated statistically since their exact placement has a large effect on the power distribution.[1] Any very reflective walls present in the building should be included in

[1] Simplified ray-tracing arguments can be used to estimate when

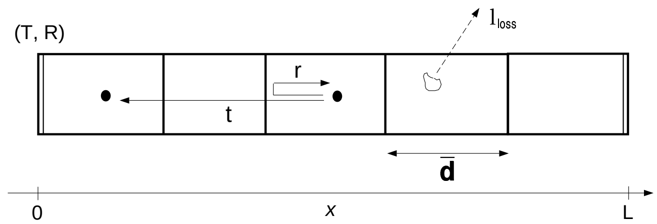

Fig. 1. Schematic of the one-dimensional propagation problem. Interior walls have a mean spacing $\bar{d}$ and reflection (transmission) coefficient r (t); the two exterior walls have reflection (transmission) coefficient R (T). In addition to scattering from the walls, scattering from local clutter causes signal to leave the building and is included statistically through the mean-free-path $l_{\rm loss}$.

a deterministic collision integral which eventually can be treated as an internal boundary condition in the statistical model. Second, a statistical approach requires a certain amount of scattering to be accurate. If the situation is predominantly line of sight, for instance, a statistical approach may be poor. Thus our models are not well adapted to pico-cell situations in which a transmitter covers only a few rooms.

By making relaxation-time approximations for the various physical scattering processes, one can define a variety of statistical scattering models. We now develop a few of these in more detail, first for the unrealistic but instructive case of one-dimensional buildings, and then for two and three dimensional buildings.

A. One-dimensional Propagation

For propagation in a one-dimensional (linear) building embedded in three-dimensional space, as pictured in Fig. 1, there are two important scattering processes: scattering from the walls and the scattering from local clutter which causes signal to exit the building. For the purposes of propagation within the building, this latter process is equivalent to actual absorption or loss within the building.

In a Boltzmann equation approach, one could treat the scattering from the walls exactly using a collision integral like Eq. (2) and treat the local clutter scattering statistically. Here we simplify, however, by making a relaxation-time approximation for each of these processes, introducing a wall mean-free-path $l_{\rm wall}$ and a mean-free-path for scattering out of the building $l_{\rm loss}$. This treatment of the scattering from the walls has two advantages: on the one hand it means that an exact description of the building floor plan is not required, and on the other hand it decreases the computational difficulty of the problem. In terms of the reflection probability from a typical wall, r, and the mean distance between the walls, $\bar{d}$, $l_{\rm wall}$ is defined by

$$l_{\rm wall} \equiv \bar{d}/r \; . \qquad (4)$$

In introducing a single wall mean-free-path we have assumed that the transmission properties of the various walls are similar. If this is not the case, one can introduce a spatially dependent mean-free-path and so treat different parts of the building, including in principle very inhomogeneous buildings with large atriums and courtyards. For simplicity of presentation, however, here we consider the homogeneous case of a single mean-free-path.

In one-dimension, the distribution function consists of either right-moving signal $f^+(x,t)$ or left-moving signal $f^-(x,t)$. In terms of these functions, the Boltzmann equation is

$$\frac{\partial f^\pm}{\partial t} \pm c\frac{\partial f^\pm}{\partial x} = -\frac{f^\pm - f^\mp}{l_{\rm wall}/c} - \frac{f^\pm}{l_{\rm loss}/c} + \frac{P_0}{2/c}\delta(x-x_0) \; . \quad (5)$$

The left-hand side is the free-space propagation; c is the speed of light. In addition to the scattering terms— one for the wall scattering and one for the loss from local clutter— we have included on the right-hand side a point source transmitter with power P_0. Multiple antennas can be easily treated using a set of delta- functions. Note that the outgoing distribution, $f_{\rm out}$ of Eq. (3), is f^- for an in-coming f^+ in the case of scattering from a wall— the signal is reflected so that the velocity reverses— and is identically zero for the case of scattering out of the building. The boundary condition to be used in solving this equation is that there is no signal coming into the building from outside; for a building of length L

$$f^+(0) = 0 \; , \qquad f^-(L) = 0 \; . \qquad (6)$$

The free-space propagation and the scattering from the walls can be combined into a single diffusive term, resulting in a simpler expression for computation and analysis. Defining $f(x) \equiv f^+(x) + f^-(x)$, one finds that the pair of Boltzmann equations (5) in the static limit ($\partial f^\pm/\partial t{=}0$) is equivalent to the single diffusion equation

$$D\frac{d^2 f}{dx^2} - \frac{f}{l_{\rm loss}} + P_0\delta(x-x_0) = 0 \; . \qquad (7)$$

Here the diffusion constant D is related to the mean-free-paths by

$$D \equiv \left[\frac{2}{l_{\rm wall}} + \frac{1}{l_{\rm loss}}\right]^{-1} \approx \frac{l_{\rm wall}}{2} \qquad (8)$$

where the last relation is valid when scattering from the walls is stronger than scattering from local clutter, which is the usual situation. The boundary condition for the diffusion equation can be derived from that for the Boltzmann equation. We allow for the fact that the exterior walls of the building are usually of a different material from the interior walls: let T and R be the transmission and reflection probabilities for the exterior walls while t and r are those for the interior walls. Then one can show that the boundary condition on $f(x)$ is

$$\frac{df}{dx} = \mp\frac{K_{\rm ext}}{l_{\rm wall}} f \; , \qquad K_{\rm ext} \equiv \frac{2T}{1+R(t-r)} \; , \qquad (9)$$

where the upper (lower) sign is for the right (left) boundary of the building. It is also possible to include the effect

the exact placement of a wall should be included [18]. In the one-dimensional case, by comparing the magnitude of the signal on the two sides of a wall, one finds that $t > \bar{d}/l_{\rm loss}$ is needed in order to use a statistical model.

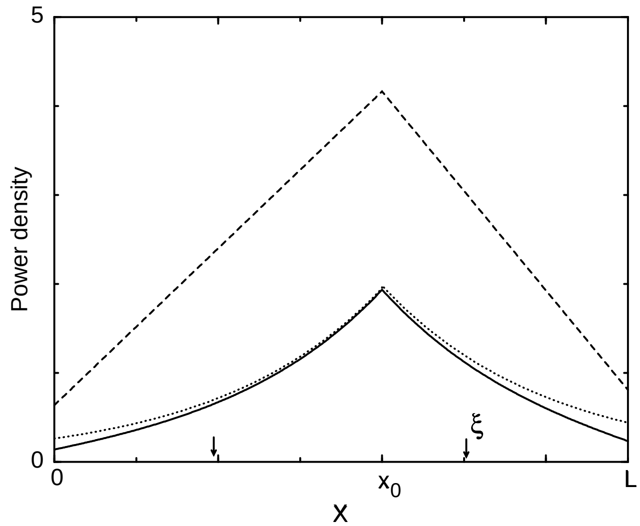

Fig. 2. Power as a function of distance for the one-dimensional diffusion model in two simple cases with the transmitter at x_0. (1) $l_{\rm loss} \to \infty$, dashed line. In the absence of local clutter scattering, the power decays linearly and the effect of the boundary extends throughout the building. (2) $l_{\rm loss} < \infty$, solid line. The power decays approximately exponentially in the interior; a purely exponential decay is indicated by the dotted line. The effect of the boundary extends a distance $\xi \equiv \sqrt{l_{\rm loss} D}$ into the building; the arrows mark a distance ξ from each boundary.

of an exceptional interior wall through a similar boundary condition; the combination of transmission coefficients which enters in this case, $K_{\rm int}$, is slightly different from $K_{\rm ext}$ given above [18]. In this way, the complex problem of propagation through the exact configuration of walls and clutter has been reduced, with approximations, to the simple diffusion equation (7).

Before turning to more realistic two and three dimensional buildings, we give the solution of the diffusion equation in two simple cases and plot them in Fig. 2. First, suppose there is no loss— all of the signal remains in the building— so $l_{\rm loss} \to \infty$. In this case the power decays linearly in the building: it is maximum at the transmitter and zero at a distance $l_{\rm wall}/K_{\rm ext}$ beyond the boundary of the building. The power at the transmitter is

$$f(x_0) = \frac{1}{D} \frac{(x_0 + l_{\rm wall}/K_{\rm ext})(L - x_0 + l_{\rm wall}/K_{\rm ext})}{L + 2l_{\rm wall}/K_{\rm ext}} P_0 \, . \tag{10}$$

Note that the boundaries essentially determine the power throughout the building.

Second, if local clutter scattering is present in the building, the power will decay exponentially, having a form

$$f(x) = a e^{-x/\xi} + b e^{+x/\xi} \tag{11}$$

in each region where the a's and b's are chosen to satisfy the boundary conditions. The decay length ξ is

$$\xi \equiv \sqrt{l_{\rm loss} D} = \frac{l_{\rm loss}}{\sqrt{1 + 2l_{\rm loss}/l_{\rm wall}}} \le l_{\rm loss} \, . \tag{12}$$

In the usual case of $l_{\rm loss} > l_{\rm wall}$, the decay length is approximately the geometric mean of the two mean-free-paths. As illustrated in Fig. 2, the presence of the boundaries can be felt for a distance ξ into the building; otherwise f is essentially given by the single exponential appropriate for an infinite lossy medium.

B. Two-dimensional Propagation

For two-dimensional propagation we have in mind propagation on a single floor of a building. Compared to the one-dimensional case, an additional physical effect is present: the transmission through the walls depends on angle for many building materials [2], [4]. Since the walls are preferentially arranged at right angles to each other, the angular dependence of the distribution function should be kept: $f = f(\mathbf{r}, \theta)$ where θ is the angle the velocity makes with respect to the x-axis. We consider the common situation in which the walls of the building are arranged along the x and y axes.

Three types of scattering processes are relevant. First, the scattering from the walls is angular dependent and assumed to be specular. Thus, this scattering couples $f(\theta)$ to the f for the three angles related by x and/or y reflection. Second, scattering from local-clutter can cause power to be deflected into the third dimension; this is equivalent to loss, as before. Third, local clutter can cause scattering from θ into an arbitrary angle within the floor considered. This scattering mixes the different angles and relaxes the distribution to the angular average of f. By using successive approximations for the effects of these scattering processes, we develop a range of models for two-dimensional propagation along the same lines as for one-dimensional buildings above.

In the Boltzmann equation approach, one could treat the scattering from the walls deterministically based on the exact location of the walls, and treat the other two processes probabilistically using a relaxation-time approximation. As before $l_{\rm loss}$ is the mean-free-path for loss. For the third scattering process which mixes angles, we call the mean-free-path l^* and the outgoing distribution $\langle f \rangle \equiv \int d\theta f(\mathbf{r}, \theta)/2\pi$. This is the first level of models.

We further simplify by treating the wall scattering statistically as well, and so introduce a second type of model. The reason for this further simplification is two fold. First, a statistical treatment of the walls removes the necessity of exactly specifying the wall geometry, something which may be inconvenient in practice and unimportant as well for a typical coverage prediction. Second, the relaxation-time approximation reduces the computational complexity of the problem.

In the two-dimensional case, the wall mean-free-path is different in the x and y directions— the mean wall separation may differ, for instance— and depends on angle:

$$\begin{aligned} l_{\rm x,wall}(\theta) &\equiv \bar{d}_x / |\cos\theta| \, r(\theta) \\ l_{\rm y,wall}(\theta) &\equiv \bar{d}_y / |\sin\theta| \, r(\pi/2 - \theta) \, . \end{aligned} \tag{13}$$

In this way, we obtain a two-dimensional statistical Boltzmann equation model for single floor propagation:

$$\hat{\theta} \cdot \nabla f(\mathbf{r}, \theta) = -\frac{f(\mathbf{r}, \theta) - f(\mathbf{r}, \pi - \theta)}{l_{\rm x,wall}(\theta)} - \frac{f(\mathbf{r}, \theta) - f(\mathbf{r}, -\theta)}{l_{\rm y,wall}(\theta)}$$

$$-\frac{f(\mathbf{r},\theta)}{l_{\rm loss}} - \frac{f(\mathbf{r},\theta)-\langle f\rangle}{l^*} + \frac{P_0}{2\pi}\delta(\mathbf{r}-\mathbf{r}_0) \quad (14)$$

where $\hat{\theta}$ is a unit vector in the direction of propagation θ. As for all our Boltzmann equations, the boundary condition to use here is that there is no flux incident on the building from the outside.

As in the one-dimensional case above, one can arrive at a third type of model by making a further approximation and pass to a diffusion equation. This is mainly done for computational reasons: the reduction in the number of variables greatly simplifies the computation. To do this, we let $\tilde{f}(\mathbf{r},\theta)$ be the sum of the f for the four angles related by x and/or y reflection where now $\theta \in [0,\pi/2]$. If the loss caused by local clutter is not too strong, f is smooth on lengths of order $l_{\rm wall}$. In this case, one can derive a diffusion equation for $\tilde{f}$,

$$D_x(\theta)\frac{\partial^2 \tilde{f}}{\partial x^2} + D_y(\theta)\frac{\partial^2 \tilde{f}}{\partial y^2} - \frac{\tilde{f}}{l_{\rm loss}} - \frac{\tilde{f}-\langle\tilde{f}\rangle}{l^*} + P_0\delta(\mathbf{r}-\mathbf{r}_0) = 0\,, \quad (15)$$

where the diffusion constants depend on angle,

$$D_x(\theta) = \cos^2\theta\left[\frac{2}{l_{\rm x,wall}(\theta)} + \frac{1}{l_{\rm loss}} + \frac{1}{l^*}\right]^{-1} \quad (16)$$

$$D_y(\theta) = \sin^2\theta\left[\frac{2}{l_{\rm y,wall}(\theta)} + \frac{1}{l_{\rm loss}} + \frac{1}{l^*}\right]^{-1}$$

We have used a single isotropic transmitter at a single point $\mathbf{r}_0$, but multiple transmitters with more complicated angular or spatial patterns could be included by using the appropriate function instead of the delta function. The boundary condition that should be used with this diffusion equation is simply that the normal derivative of $\tilde{f}(\mathbf{r},\theta)$ at the boundary obey the same equation as in one-dimension, Eq. (9). In this way the fact that exterior walls often have very different transmission properties from interior walls is simply included. Particularly reflective internal walls can be included as an internal boundary condition of a similar type. Note that because of the angular dependence of the wall scattering, the θ dependence of $\tilde{f}(\mathbf{r},\theta)$ must be retained and the resulting equation is highly anisotropic.

For the fourth and simplest level of model in our range, we further simplify by neglecting both the anisotropy and the boundary conditions: this is the case of lossy diffusion in a *bulk* isotropic medium. The angle mixing scattering characterized by l^* is no longer relevant because of isotropy. In this case, the solution of the diffusion equation with the transmitter at the origin is

$$f(|\mathbf{r}|) = \frac{P_0}{2\pi D}K_0(|\mathbf{r}|/\xi) \sim \frac{P_0}{2\pi D}\sqrt{\frac{\pi\xi}{2|\mathbf{r}|}}e^{-|\mathbf{r}|/\xi} \quad (17)$$

where K_0 is the modified Bessel function [19] and ξ is the decay length in (12). The approximate expression on the right is valid for large $|\mathbf{r}|$. This expression is a simple functional form depending on a single parameter ξ and so is conceptually similar to empirical power-law decay that has been considered in the past. Of course, boundary conditions could be neglected but the anisotropy of the medium retained, resulting in a slightly different bulk anisotropic model.

C. Three-dimensional Propagation

The generalization from two-dimensional to three-dimensional propagation is straight forward. No new physical effects come into play— the same three scattering mechanisms are present as in two dimensions. Thus, one can develop the same range of models as in two dimensions following basically the same procedures.

There are several differences of detail in three dimensions, however, which may make a significant difference in practice. First, floors and ceilings tend to be much more reflective than walls within a floor. Since the derivation of the diffusion equation from the statistical Boltzmann equation breaks down if the walls are too reflective, a three-dimensional diffusion equation may not be useful; rather, the statistical Boltzmann equation may be required. Second, local clutter can no longer scatter signal out of the building as easily. This may mean that the loss caused by local clutter is much smaller and that one may only need angle mixing scattering characterized by l^*.

This completes the presentation of the range of models. All of the models are for the mean power— wave-interference effects are neglected— and are conceived within the geometrical optics approximation to the full wave propagation, though the statistical scattering approach may be able to capture some diffractive effects. We started from a deterministic Boltzmann equation in which the scattering properties of all of the obstacles was specified, an approach equivalent to ray tracing. Successive approximations led to first a statistical Boltzmann equation, then a diffusion equation with boundary conditions, and finally a bulk diffusion equation whose solution is a simple functional form.

III. Results and Comparison to Measurements

In order to indicate the usefulness of the models introduced above, we compare two of them to measurements of propagation in several buildings. The first model we implement is the two-dimensional diffusion equation with boundary conditions, given by (15). The second is the bulk diffusion model (17) which is just an isotropic decay law. Both of these models are on the "simple" side of our range of models. We have chosen these two models for ease of implementation and to emphasize the potential simplicity of the approach.

The measurements used were obtained by O. Landron, R. Valenzuela, and co-workers, and have been previously reported in Refs. [20], [11]. We use data at 1.9 GHz from five different areas in three Lucent Technologies or AT&T buildings: a small sheetrock office building (Crawford Hill), a large sheetrock conference room area (Learning Center— conference), a sheetrock overnight accommodation area (Learning Center— rooms), a metallic partition office area (Middletown— wing), and a mixed metallic par-

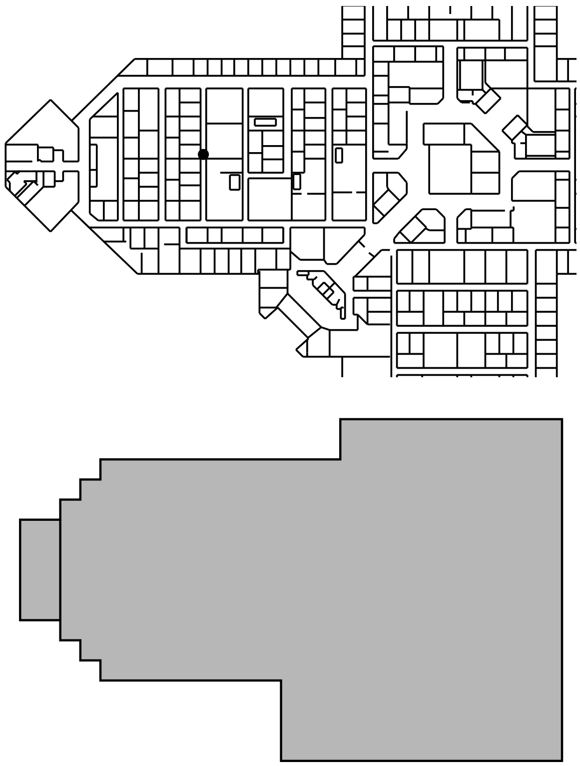

Fig. 3. Schematic of the single floor, two-dimensional propagation problem. The top panel is the floor plan for the Middletown wing building environment with the transmitter indicated by the black dot, and the bottom panel is the approximation to this floor plan used in the diffusion equation. The exact location of the walls is replaced by a diffusive medium in which the diffusion constants are related to the mean wall separation and the reflection coefficient of the most common type of wall. Local clutter scattering is included through two mean-free-paths $l_{\rm loss}$ and l^*. The exterior walls of the building, as well as certain important interior walls such as that on the left of the wing shown, are included through boundary conditions depending on their reflection and transmission coefficients.

tition and sheetrock office area (Middletown— center). For each of these environments, we have implemented the two-dimensional diffusion equation model in a building of the appropriate shape.

None of the building environments used in our comparisons have a large inhomogeneity— there are no atriums or courtyards. This is not a restriction of the models: the statistical Boltzmann model could handle such a situation by using a spatially varying mean-free-path, though the bulk diffusion approach could not. However, for simplicity and because of limited data available we do not study such a case here. Several of the buildings include less dramatic inhomogeneity: the Learning Center conference area is formed of mid-sized conference rooms with smaller rooms around, the Middletown-wing area has an interior reinforced concrete wall which is included as a boundary condition [see Fig. 3 and the discussion after (10)], and the metallic partitions in the otherwise sheetrock Middletown-center area are included as interior boundaries.

To illustrate the input parameters involved, Fig. 3 shows a floor plan of one of the building environments used as well as the form it takes for the diffusion equation. The interior walls are replaced by an anisotropic diffusive medium. The mean wall separation taken from the floor plan combined with the reflection coefficient for the typical wall type gives the wall mean-free-path via (13). The reflection coefficient as a function of angle is obtained from a layered dielectric model [21] for the most common type of wall. The layered dielectric model is also used to find the reflection coefficient for the exterior walls and any exceptional interior walls which should be included explicitly (such as those made of reinforced concrete).

The remaining two parameters in the diffusion model (15) are the mean-free-paths $l_{\rm loss}$ and l^*. These two parameters must be determined empirically since they are related in a complicated way to the contents of the building. Similarly, for the bulk diffusion model, the decay length ξ must be determined empirically. The use of empirical parameters in indoor propagation is widely practiced and accepted; for instance, in the power-law decay approach the exponent is chosen empirically [4], [5], [6], and in ray-tracing empirical loss or material parameters are introduced [7], [9], [11]. The way in which these parameters are chosen is described in Section A below.

Once the parameters are specified, the solution of the boundary diffusion model proceeds using a standard numerical approach. First, the partial differential equation (15) is discretized on a spatial mesh which covers the building. The mesh spacing is approximately the mean room size since this is the length scale for mean intensity variation. Note that this is much larger than the carrier wavelength, the scale of the wave-interference effects that we neglect. Second, the angle is discretized; typically we use eight θ in $[0, \pi/2]$. Discretization results in a large, but very sparse, set of linear equations. Third, the boundary conditions for both external and internal walls [Eq. (9)] are discretized and incorporated in the set of linear equations. Finally, the system of equations is solved using standard sparse matrix techniques.

The computation of propagation in this way is reasonably efficient in that it grows as a small power of the system size (between 1 and 2). Because all multiple reflections within the diffusive medium are included in this approach, these models are particularly well suited to large buildings in which in addition paths with many scattering events are important. In contrast, ray-tracing approaches are often limited to a small number of reflections per trajectory n_r for two reasons. First, the number of trajectories grows exponentially with n_r, and, second, the computation grows as a power of the system size where the power is n_r (typically ≥ 2).

A. Empirical Parameters

In order to fix the empirical parameters we proceed in one of two ways. First, we simply fit the calculated result of the diffusion models to the entire measured data set in each building. This then tells us the best we can possibly do: if the error statistics in this case do not meet the target values (mean less than 3 dB and standard deviation less than 6 dB) then one needs a more accurate model. The disadvantage of this approach is that it gives no information about the predictive accuracy of the model in a different building.

Our second way to determine the mean-free-paths is to fit the prediction to a small number of measured points chosen randomly, typically 10 points. Then we can use these parameters to calculate the signal at all the other measured points and so get some sense of the predictive accuracy of the model. We call this way of proceeding "sample mode".

In comparing with the measurements, we will only consider the parts of the buildings far from the transmitter. This is the region of greatest practical relevance— close to the transmitter there is always sufficient power. In fact, the diffusion models tend to make a systematic error close to the transmitter: in this region the propagation is not statistical in that a few deterministic paths tend to dominate. On the other hand, this error is of little practical importance in most situations and is excluded in this way. However, one situation in which the region close to the transmitter is important is for pico-cell applications where a transmitter covers only a few rooms; for describing particular pico-cells, our diffusive models are probably not well adapted. In what follows, the region within 15 m of the transmitter has been excluded in finding the error statistics; improvements to the models could probably reduce this exclusion zone to less than 10 m.

Finally, we have found that the error statistics are much less sensitive to l^*— the angle mixing scattering process— than they are to $l_{\rm loss}$— the scattering of signal out of the floor considered or absorption. As a consequence we take $l^* \to \infty$ in what follows, and leave the question of the importance of angular mixing to future work. Thus there is one empirical parameter in both our bulk diffusion model (17)— the decay length ξ— and our diffusion model with boundary conditions (15)— the loss mean-free-path $l_{\rm loss}$.

B. Fit to Measurements

Both models were fit to the entire dataset in each of the five building environments. To get a sense of the difference between the calculation and the measurement, we plot in Fig. 4 the ratio of the calculated power $P_{\rm calc}$ to the measured power $P_{\rm meas}$ in dB versus the distance from the transmitter. For this figure we have used the diffusion equation with boundary conditions and the Middletown-wing building environment shown in Fig. 3. Three features of the plot stand out. First, the scatter of the error indicates that the propagation environment is not homogeneous and isotropic; if this were the case, the measurements would be a function of distance only. Second, the diffusion model under-predicts the power at short distance where it is expected to be poor. Finally, by fitting the loss parameter, we obtain the mean behavior at large distance correctly. The scatter in our error statistics at large distance (30-40 m) is typical for propagation prediction (such as ray-tracing) in this range; the larger the distance the more difficult and uncertain the prediction.

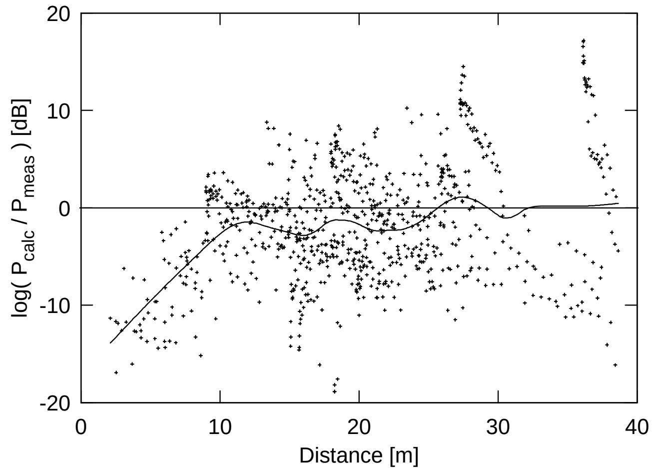

Fig. 4. Comparison of calculated and measured power as a function of distance from the transmitter in the wing of the Middletown building. The points are the log of the ratio of calculated to measured power in dB, and the line is a guide to the eye. The calculation uses the two-dimensional diffusion equation with boundary conditions and the optimal value of $l_{\rm loss}$. The calculation is poor in the unimportant region close to the transmitter but finds the important long distance behavior correctly.

To summarize succinctly the quality of the calculation, we give in Table I the mean and standard deviation of the error statistics $[\log(P_{\rm calc}/P_{\rm meas})]$ for all points further than 15 m from the transmitter, as well as the value of the fitted parameter used. Values of the fit parameters correspond to a characteristic decay length several times the typical size of a room, which seems reasonable. Note that the value of the fit parameter varies more widely for the bulk diffusion model than for the model including boundary conditions, suggesting that the inclusion of boundaries is important.

The most important conclusion from Table I, is that both diffusion models meet our target values for the error statistics when the empirical parameter is fit to the entire data set. This is somewhat surprising for the bulk diffusion model which is, after all, a very simple isotropic decay law, and suggests that a simple power-law decay may also do well. In this regard, we caution that none of the buildings we consider have large inhomogeneities, and one may expect that the bulk diffusion model will do poorly in such situations while the boundary diffusion model may do nearly as well. In any case, for the examples used here, we can go on to see if either model has predictive value.

C. Sample Mode

In this mode, we choose N points at random from among the measured values farther than 15 m from the transmitter and fit the empirical parameter of each model to these N points. Then the calculation using this parameter is compared to the other measured points, and the quality is evaluated using the mean and standard deviation of the error statistics. Finally, the set of N points is varied randomly, such that the procedure is carried out 100 times. The result is a distribution of the mean error and standard deviation over the 100 samples.

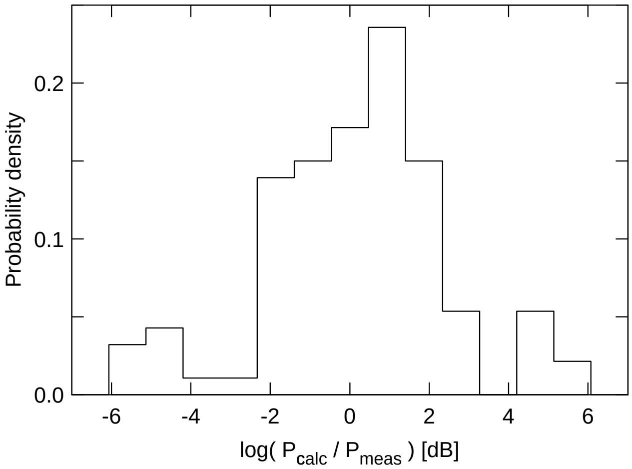

Fig. 5. Histogram of the mean of the error statistics in sample mode, as the sample of points is varied ($N = 10$). The mean is taken with respect to all the other measured data points; the Middletown wing building is used. Note that this distribution is centered near zero, but the width may be significant.

In all the cases we studied, we found that the distribution of the standard deviation is very narrow. The mean of this distribution is close to the standard deviation for a fit to the entire dataset as in the last section. In contrast, the distribution of the mean has a significant width; Fig. 5 shows this distribution for $N = 10$ in the case of the diffusion equation with boundaries in the Middletown-wing case. The additional error coming from sampling such a small number of points is approximately 2 dB, which is significant compared to our target value of 3 dB.

As the number of sampling points is increased, the distribution of the mean error narrows and becomes Gaussian. Note that we have made no attempt to optimize the sampling strategy, and it is likely that one can substantially decrease the additional sampling error by carefully choosing points which are distant from each other and from the transmitter.

The error statistics [$\log(P_{\rm calc}/P_{\rm meas})$] for all five buildings are given in Table II. The results are within our target values for both models in all five cases. Two of the buildings are long and thin— Crawford Hill and the rooms part of the Learning Center— and in these cases the inclusion of boundary conditions improves the error statistics as one would expect. In one building— Middletown atrium— the fluctuations of the fit parameter are particularly large but the error statistics nonetheless remain good. While the fluctuations could undoubtedly be reduced by better sampling, this indicates that the results are relatively insensitive to the amount of loss, being dominated by the scattering from the walls.

The conclusion from these results is that if one is willing to make a few measurements in a building in order to fix one empirical parameter, one can make a very good prediction for propagation throughout the building using a surprisingly simple model. Again we caution that these results may be overly optimistic in the case of the bulk diffusion model since none of the buildings we consider have large inhomogeneities, a point to which we return below.

IV. Conclusions

By treating the scattering of the signal by obstacles in the building probabilistically, we have introduced a range of models for wireless propagation. The starting point is a deterministic Boltzmann equation for a completely defined geometry which is completely equivalent to ray-tracing. The range of models includes (1) a Boltzmann equation in which some of the scattering is deterministic but some is treated probabilistically, (2) a statistical Boltzmann equation in which all of the scattering is treated probabilistically, (3) an anisotropic diffusion equation in the region of the building, and, finally, (4) a bulk diffusion equation (either isotropic or anisotropic) which yields a simple decay law. At one end of this range one uses a very complex description of the environment to obtain an accurate model of the propagation. At the other end of this range one uses a very simple description of the environment but obtains a less accurate model of the propagation. As one moves through the range, then, one trades off the required *complexity* of the building description for the *accuracy* of the prediction. In any given application, one can choose what trade-off to make by using the appropriate model from the range.

The models are quite flexible in principle. Buildings of a variety of shapes and with inhomogeneous or anisotropic features can be treated. Specific structural features which have a strong influence on the propagation can be included as an internal boundary condition. Multiple or extended transmitters can be incorporated.

We have implemented two of these models— the two-dimensional (single floor) diffusion model with boundary conditions and the bulk isotropic diffusion model— and compared them to previously obtained [20], [11] measurements. Both models provide a very good fit to the data when one optimizes the loss parameter. Thus these models potentially provide a good description of the propagation. In order to evaluate the models more rigorously, we use a sampling procedure to fix the empirical loss parameter. Such a procedure could be used for any of the commonly used propagation models since they all involve an empirical parameter. We find that a few samples is sufficient to fix the loss parameter in both models and give good agreement with the entire measured data set. Thus these models do have predictive value.

The dataset that we have used for evaluating the models consists of five fairly standard office building environments, but it is important to keep in mind that while several of the buildings are somewhat inhomogeneous (the conference part of the Learning Center, for instance), the dataset does not include any buildings with very large inhomogeneities. In such a building— one with an atrium, courtyard, or large auditorium, for instance— a prediction using a simple decay law as in the bulk diffusion model is unlikely to work: different areas of the building should be treated differently and then joined together. Such large inhomogeneity can be treated naturally in either the boundary diffusion model or the statistical Boltzmann equation using a spatially varying mean-free-path or internal boundary conditions, and, of course, can be treated using ray-tracing as well.

We leave two outstanding questions for future work. The first is how exactly to make the trade-off between complexity and accuracy and so choose which model to use. This will involve a more extensive comparison of the different models, including the standard ones of ray-tracing and power-law decay, both with the data and with each other in terms of computational efficiency. The second question is whether these models are predictive in an *a priori* mode: whether the loss parameter can be determined for a certain building type, for instance, and then used to model propagation in other buildings of that type without further measurements. Again, answering this question will involve extensive comparison with measurements.

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References

[1] D. Molkdar, "Review on radio propagation into and within buildings," IEE Proc., vol. 138, pp. 61-73, 1991.
[2] H.L. Bertoni, W. Honcharenko, L.R. Maciel, and H.H. Xia, "UHF propagation prediction for wireless personal communications," Proc. of the IEEE, vol. 82, pp. 1333-1359, Sep. 1994.
[3] J.B. Andersen, T.S. Rappaport, and S. Yoshida, "Propagation measurements and models for wireless communications channels," IEEE Communic. Mag., pp. 42-49, Jan. 1995.
[4] T.S. Rappaport, *Wireless Communications*, Prentice Hall, 1996, pp. 69-196.
[5] A.A.M. Saleh and R.A. Valenzuela, "A statistical model for indoor multipath propagation," IEEE J. Select. Areas in Commun., vol. JSAC-5, pp. 128-137, Feb. 1987.
[6] S.Y. Seidel and T.S. Rappaport, "914 MHz path loss prediction models for indoor wireless communications in multifloored buildings," IEEE Trans. Antennas Propagat., vol. 40, pp. 207-217, Feb. 1992.
[7] W. Honcharenko, H.L. Bertoni, J.L. Dailing, J. Qian, and H.D. Yee, "Mechanism governing UHF propagation on single floors in modern office buildings," IEEE Trans. Veh. Technol., vol. 41, pp. 496-504, Nov. 1992.
[8] R.A. Valenzuela, "A ray tracing approach to predicting indoor wireless transmission," in Proc. IEEE Vehicular Technology Conf. (Secaucus NJ, 1993), pp. 214-218.
[9] S.Y. Seidel and T.S. Rappaport, "Site-specific propagation prediction for wireless in-building personal communication system design," IEEE Trans. Veh. Technol., vol. 43, pp. 879-891, Nov. 1994.
[10] W. Honcharenko, H.L. Bertoni, and J.L. Dailing, "Bilateral averaging over receiving and transmitting areas for accurate measurements of sector average signal strength inside buildings," IEEE Trans. Antennas Propagat., vol. 43, pp. 508-511, May 1995.
[11] S.J. Fortune, D.M. Gay, B.W. Kernighan, O. Landron, R.A. Valenzuela, and M.H. Wright, "WISE design of indoor wireless systems: Practical computation and optimization," IEEE Computational Sci. Eng., pp. 58-68, Spring 1995.
[12] G.E. Athanasiadou, A.R. Nix, and J.P. McGeehan, "Indoor 3D ray tracing predictions and their comparison with high resolution wideband measurements," in Proc. IEEE Vehicular Technology Conf. (Atlanta GA, 1996), pp. 36-40.
[13] H.H. Xia, A.B. Herrera, S. Kim, and F.S. Rico, "A CDMA-distributed antenna system for in-building personal communications services," IEEE J. Select. Areas Commun., vol. 14, pp. 644-650, May 1996.
[14] J.H. Tarng, W.R. Chang, and B.J. Hsu, "Three-dimensional modeling of 900-MHz and 2.44-GHz radio propagation in corridors," IEEE Trans. Veh. Tchnol., vol. 46, pp. 519-527, May 1997.
[15] F. Reif, *Fundamentals of Statistical and Thermal Physics*, McGraw-Hill, 1965, pp. 494-525.
[16] R. Courant and D. Hilbert, *Methods of Mathematical Physics*, Interscience, 1962, p. 28.
[17] N.W. Ashcroft and N.D. Mermin, *Solid State Physics*, Holt Rinehart and Winston, 1976.
[18] D. Ullmo and H.U. Baranger, unpublished.
[19] M. Abramowitz and I.E. Stegun, *Handbook of Mathematical Functions*, U.S. Dept. of Commerce, 1964, pp. 374-379.
[20] R.A. Valenzuela, O. Landron, and D.L. Jacobs, "Estimating local mean signal strength of indoor multipath propagation," IEEE Trans. Veh. Technol., vol. 46, pp. 203-212, Feb. 1997.
[21] M. Born and E. Wolf, *Principles of Optics*, Pergamon, 1980.

TABLE I

Error statistics for fit to measured data in five building environments. Each entry consists of the mean error (in dB), the standard deviation (in dB), and the value of the fitting parameter— ξ for the bulk model and $l_{\rm loss}$ for the boundary model (in m).

	Learning Ctr. conference			Crawford Hill			Learning Ctr. rooms			Middletown atrium			Middletown wing		
Bulk Diffusive	-0.8	5.0	19	2.1	4.9	53	1.9	7.0	15	-0.6	5.4	40	-0.4	6.2	20
Boundary Diffusive	-0.5	5.7	17	0.8	3.0	17	1.2	4.6	7	-0.6	5.0	40	-1.2	6.4	29

TABLE II

Comparison with measured data using sample mode (N= 10). The mean and standard deviation over the set of different sampling points is given for the mean error (in dB), the standard deviation (in dB), and the fitting parameter— ξ for the bulk model and $l_{\rm loss}$ for the boundary model (in m).

		Learning Ctr. conference		Crawford Hill		Learning Ctr. rooms		Middletown atrium		Middletown wing	
Bulk Diffusive	mean	-0.6	1.4	1.4	1.7	1.5	2.3	-0.6	1.8	-0.2	2.0
	std. dev.	4.5	0.3	5.2	0.6	7.2	0.6	5.4	0.3	6.2	0.2
	ξ	24	3	50	10	14	2	40	11	21	4
Boundary Diffusive	mean	-1.2	1.5	0.8	1.0	0.7	1.7	-0.5	1.8	0.2	2.4
	std. dev.	5.3	0.3	3.1	0.3	5.3	0.3	5.6	0.2	6.4	0.1
	$l_{\rm loss}$	18	2	17	1	7	1	43	11	32	7