ELSEVIER

Physica E 1 (1997) 268–273

PHYSICA E

Semiclassical approach to orbital magnetism of interacting diffusive quantum systems

D. Ullmo [a,b], K. Richter [c,*], H.U. Baranger [a], F. von Oppen [d], R.A. Jalabert [e]

[a] *Bell Laboratories-Lucent Technologies, 700 Mountain Avenue, Murray Hill, NJ 07974-0636, USA*
[b] *Division de Physique Théorique, Institut de Physique Nucléaire, 91406 Orsay Cedex, France*
[c] *Max-Planck-Institut für Physik komplexer Systeme, Nöthnitzer Str. 38, 01187 Dresden, Germany*
[d] *Department of Condensed Matter Physics, Weizmann Institute of Science, 76100 Rehovot, Israel*
[e] *Université Louis Pasteur, IPCMS-GEMME, 23 rue du Loess, 67037 Strasbourg Cedex, France*

Abstract

We study interaction effects on the orbital magnetism of diffusive mesoscopic quantum systems. By combining many-body perturbation theory with semiclassical techniques, we show that the interaction contribution to the ensemble-averaged quantum thermodynamic potential can be reduced to an essentially classical operator. We compute the magnetic response of disordered rings and dots for diffusive classical dynamics. Our semiclassical approach reproduces the results of previous diagrammatic quantum calculations.

PACS: 03.65.Sq; 05.45.+b; 05.30.Fk; 73.20.Dx

Keywords: Semiclassical theory; Interacting electrons; Disordered systems; Orbital magnetism

1. Introduction

The interplay of disorder and interactions in mesoscopic systems has attracted considerable attention [1]. Interaction effects on transport through small quantum dots [2, 3] as well as on thermodynamic properties like persistent currents and orbital magnetism are of present interest. In the latter case, the unexpectedly large measured persistent current of small metal rings [4–6] pointed towards the importance of such interaction effects and motivated a large number of theoretical approaches [7, 8].

For the description of thermodynamic quantities, semiclassical expansions have proven particularly useful, both within the independent-particle model [9–13] and for interaction effects [14, 15]. These studies established a close relation between the classical dynamics and the quantum-mechanical magnetic response. In particular, studies of ballistic systems showed that the quantum thermodynamic properties are sensitive to whether the classical dynamics is regular or chaotic [10–13, 15].

In this paper we apply these semiclassical techniques to the orbital magnetism of interacting sys-

* Corresponding author. Tel.: +49 351 871 2210; fax: +49 351 871 1199; e-mail: richter@mpipks-dresden.mpg.de.

1386-9477/97/$17.00
PII S1386-9477(97)00058-1

tems whose non-interacting classical dynamics is *diffusive*. Specifically, we present semiclassical derivations of the interaction contributions to the persistent current of metal rings and to the susceptibility of singly connected two-dimensional diffusive systems. We recover results obtained previously by quantum diagrammatic calculations [16–20], showing that the semiclassical approach is on the same level of approximation. By semiclassically evaluating the relevant diagrams appearing in the many-body perturbation series for the thermodynamic potential, we express the latter in terms of an essentially classical operator. This expression provides a convenient starting point for further calculations. Moreover, by making the connection with the classical dynamics, it provides a physically intuitive picture of the interplay between disorder and interaction.

2. Diagrammatic perturbation theory

We are interested in the orbital magnetism of a mesoscopic quantum system subject to an external magnetic field B. While the magnetic response of a singly connected system is usually measured in terms of its susceptibility χ, the magnetic moment of a ring-type structure threaded by a flux $\phi = BA$ (where A is the enclosed area) is usually described by the related persistent current I. Both are given in terms of the thermodynamic potential Ω as (V being the area (volume) of the structure)

$$I \equiv -c\frac{\partial \Omega}{\partial \phi}; \quad \chi \equiv -\frac{1}{V}\frac{\partial^2 \Omega}{\partial B^2}. \tag{1}$$

To calculate the interaction contribution to the magnetic response, the high-density expansion (RPA) of the thermodynamic potential [21] has to be extended by including interaction corrections from diagrams with the Cooper channel. This was originally performed in the context of superconducting fluctuations and then applied to disordered normal metals [16–20] Such expansions usually yield reliable results even beyond the high-density limit, if the relevant sets of terms are properly resummed. The relevant Cooper-like diagrams are shown in Fig. Fig. 1. The screened Coulomb interaction (wavy lines) can be treated as local [18, 19]: $U(\boldsymbol{r}-\boldsymbol{r}') = \lambda_0 N(0)^{-1}\delta(\boldsymbol{r}-\boldsymbol{r}')$. Here, $N(0)$ denotes the density of states and the bookkeeping index $\lambda_0 = 1$ identifies the order of perturbation. For the local interaction, direct and exchange term are equivalent up to a factor of (-2) due to the spin sums and the different number of fermion loops. The corresponding perturbation expansion for this interaction contribution Ω to the thermodynamic potential, which yields the magnetic response, can be formally expressed as [17, 18]

$$\begin{aligned}\Omega &= -\frac{1}{\beta}\sum_{n=1}^{\infty}\frac{(-\lambda_0)^n}{n} \\ &\quad \times \sum_{\omega}\int \mathrm{d}\boldsymbol{r}_1 \ldots \mathrm{d}\boldsymbol{r}_n\, \Sigma_{\boldsymbol{r}_1,\boldsymbol{r}_2}(\omega)\ldots\Sigma_{\boldsymbol{r}_n,\boldsymbol{r}_1}(\omega) \\ &= \frac{1}{\beta}\sum_{\omega}\mathrm{Tr}\{\ln[1+\lambda_0\hat{\Sigma}(\omega)]\}.\end{aligned} \tag{2}$$

Here, ω denotes the bosonic Matsubara frequencies $\omega = 2\tilde{n}\pi/\beta$ with $\beta = 1/kT$. The particle–particle propagator $\hat{\Sigma}(\omega)$ is expressed (in position representation) in terms of products of finite-temperature Green's functions as [21]

$$\Sigma_{\boldsymbol{r},\boldsymbol{r}'}(\omega) = \frac{1}{\beta N(0)}\sum_{\varepsilon}^{E_F}\mathscr{G}_{\boldsymbol{r},\boldsymbol{r}'}(\varepsilon)\mathscr{G}_{\boldsymbol{r},\boldsymbol{r}'}(\omega-\varepsilon). \tag{3}$$

Here, the sum runs over the fermionic Matsubara frequencies $\varepsilon = (2n+1)\pi/\beta$. The short-length (high-frequency) behavior is included in the screened interaction, thus requiring a cutoff of the frequency sums at the Fermi energy E_F [18]. The straight lines in Fig. 1 represent finite-temperature Green's functions of the non-interacting system. They are of the form

$$\mathscr{G}_{\boldsymbol{r},\boldsymbol{r}'}(\varepsilon) = \theta(\varepsilon)G^{R}_{\boldsymbol{r},\boldsymbol{r}'}(E_F+\mathrm{i}\varepsilon) + \theta(-\varepsilon)G^{A}_{\boldsymbol{r},\boldsymbol{r}'}(E_F+\mathrm{i}\varepsilon) \tag{4}$$

in terms of the retarded and advanced Green's functions $G^{R,A}$ which are related by $G^{A}_{\boldsymbol{r},\boldsymbol{r}'}(E) = [G^{R}_{\boldsymbol{r}',\boldsymbol{r}}(E^*)]^*$. For diffusive systems, they include the presence of the disorder potential.

3. Semiclassical formalism

Both in ballistic and diffusive samples, the Fermi wavelength λ_F is often the shortest length scale. It is in this situation that we can apply semiclassical techniques to compute $\Sigma_{\boldsymbol{r},\boldsymbol{r}'}(\omega)$. Here, we will moreover assume that the magnetic field B is classically weak,

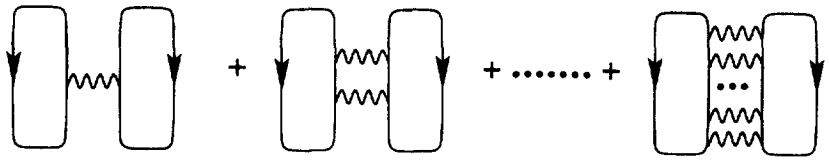

Fig. 1. Leading Cooper-channel diagrams for the interaction contribution to thethermodynamic potential.

i.e., that the cyclotron radius $R_c \gg \min\{l, L\}$ (with l the elastic mean free path and L the system size).

Semiclassically, the retarded Green's function is represented as a sum of contributions $G^{R;j}_{r,r'}$ over all classical paths j from r to r' [22],

$$G^{R}_{r,r'}(E) \simeq \sum_{j:r\to r'} D_j \, e^{iS_j/\hbar - i\pi\nu_j/2}. \tag{5}$$

Here $S_j = \int_r^{r'} \boldsymbol{p}\cdot d\boldsymbol{r}$ is the classical action of trajectory j. The prefactor D_j includes the classical phase-space density $[D_j = (1/\sqrt{2\pi(i\hbar)^3 \dot{x}\dot{x}'})|\partial^2 S_j/\partial y \partial y'|^{1/2}$ in two dimensions]. ν_j is a Maslov index. The semiclassical approximation makes the temperature and magnetic-field dependences of the finite-temperature Green's function transparent. Employing $(\partial S_j/\partial E) = t_j$ and $(\partial S_j/\partial B) = (e/c)A_j$, where t_j and A_j are the traversal time and area, one finds

$$G^{R;j}_{r,r'}(E_F + i\varepsilon, B) \simeq G^{R;j}_{r,r'}(E_F, B=0) \times \exp[-\varepsilon t_j/\hbar] \times \exp[i2\pi B A_j/\phi_0] \tag{6}$$

where $\phi_0 = hc/e$ is the flux quantum. Note that temperature exponentially suppresses the contributions of long paths to each Green's function.

Semiclassically, the particle–particle propagator $\Sigma_{r,r'}(\omega)$ is then represented as a sum over pairs of paths between r and r'. Off-diagonal pairs (of different paths) generally contain highly oscillatory contributions which do not survive an ensemble (disorder) average. (There can be exceptions as discussed in Ref. [15].) On the other hand, the diagonal pairing of each orbit j with its time reverse persists upon averaging since their dynamical phases $\exp[iS_j(B=0)/\hbar]$ cancel while retaining a magnetic-field dependence. A more detailed semiclassical analysis [15] shows that the Cooper series in Fig. 1 contains the magnetic-field sensitive contribution to Ω which is leading order in $\hbar$.

Using Eqs. (4)–(6) in Eq. (3) and performing the Matsubara sum yields for the diagonal part of $\hat{\Sigma}$

$$\Sigma^{(D)}_{r,r'}(\omega) \simeq \frac{\hbar}{\pi N(0)} \sum_{j:r\to r'}^{L_j > \Lambda_0} |D_j|^2 \frac{R(2t_j/t_T)}{2t_j} \times \exp\left[\frac{i4\pi B A_j}{\phi_0}\right] \times \exp\left[-\frac{\omega t_j}{\hbar}\right]. \tag{7}$$

The sum runs over all trajectories longer than the cutoff $\Lambda_0 = \lambda_F/\pi$ [corresponding to the upper bound E_F on the Matsubara sum in Eq. (3)]. The temperature dependence in Eq. (7) enters through the function $R(x) = x/\sinh(x)$ introducing the time scale

$$t_T = \frac{\hbar\beta}{\pi} \tag{8}$$

and the related length scale $L_T = v_F t_T$, with v_F being the Fermi velocity. This semiclassical framework allows us to reduce the original quantum problem to $\Sigma^{(D)}$, which no longer exhibits variations on the quantum scale λ_F but only on classical scales. We emphasize that the representation, Eq. (7), of $\Sigma^{(D)}$ is rather general since we have not yet made any assumption about the classical dynamics of the system. In particular, it applies to both diffusive and ballistic systems. On the basis of Eq. (7), we have recently studied interaction effects in ballistic quantum dots [15]. Specifically, we show that the interaction-induced orbital magnetism scales differently for systems with regular and chaotic non-interacting classical counterparts.

Here, we focus on diffusive systems for which it is useful to relate $\Sigma^{(D)}$ to classical probabilities satisfying the diffusion equation. To this end we introduce an additional time integration in Eq. (7) and make use of the relation [9]

$$\frac{1}{2\pi^2} \sum_{j:r\to r'} |D_j|^2 \delta(t - t_j) = \frac{N(0)}{2\pi\hbar} P(r, r'; t) \tag{9}$$

between the weights $|D_j|^2$ and the classical probability $P(\boldsymbol{r},\boldsymbol{r}';t)$ to propagate from $\boldsymbol{r}$ to $\boldsymbol{r}'$ in time t.

An nth order contribution to Ω in Eq. (2) then contains expressions for the joint return probability $P(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_n,\boldsymbol{r}_1;t_1,\ldots,t_n|A)$ to visit the points $\boldsymbol{r}_i$ (with t_i being the time between $\boldsymbol{r}_i$ and $\boldsymbol{r}_{i-1}$) under the condition that the enclosed area is A. In diffusive systems, the probability is multiplicative, namely $\int \mathrm{d}\boldsymbol{r}_1\ldots\mathrm{d}\boldsymbol{r}_n P(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_n,\boldsymbol{r}_1;t_1,\ldots,t_n|A) = \int \mathrm{d}\boldsymbol{r} P(\boldsymbol{r},\boldsymbol{r};t_{\mathrm{tot}}|A)$ with $t_{\mathrm{tot}} = \sum t_i$. The contribution to Ω in Eq. (2) from the diagonal terms $\Sigma^{(D)}$ then yields

$$\Omega^{(D)} = \sum_n \Omega_n^{(D)} = \frac{1}{\beta}\int \mathrm{d}\boldsymbol{r}\int \mathrm{d}t \coth\left(\frac{t}{t_{\mathrm{T}}}\right) K(t)\mathscr{A}(\boldsymbol{r},t;B), \quad (10)$$

where $\coth(t/t_{\mathrm{T}})$ arises from the ω-sum in Eq. (2) and

$$K(t) \equiv \sum_n K_n(t) = -\sum_n \frac{(-\lambda_0)^n}{n}\left\{\int\prod_{i=1}^{n}\left[\frac{\mathrm{d}t_i R(2t_i/t_{\mathrm{T}})}{2t_i}\right]\delta(t-t_{\mathrm{tot}})\right\} \quad (11)$$

$$\mathscr{A}(\boldsymbol{r},t;B) \equiv \int \mathrm{d}A \cos\left(\frac{4\pi BA}{\phi_0}\right) P(\boldsymbol{r},\boldsymbol{r};t|A). \quad (12)$$

$K(t)$ accounts for temperature effects while $\mathscr{A}$ contains the field dependence and the classical return probability. Eqs. (10)–(12) are a general and convenient starting point to compute the orbital response of disordered systems.

4. Diffusive rings

We start with the computation of the first-order interaction contribution, $\Omega_1^{(D)}$, to illustrate the main ideas. Consider a (thin) disordered ring of width b, cross section σ and circumference L. For $L \gg l, b$ the motion of particles around the ring effectively follows a law for one-dimensional diffusion. Since the area enclosed is given in terms of the number m of windings around the ring, one has

$$P(\boldsymbol{r},\boldsymbol{r};t|A) = \sum_{m=-\infty}^{+\infty}\frac{1}{\sigma}\frac{1}{\sqrt{4\pi Dt}}\exp\left(-\frac{m^2L^2}{4Dt}\right)\times\delta\left(A-\frac{mL^2}{4\pi}\right), \quad (13)$$

where $D = v_{\mathrm{F}}l/d$ is the diffusion constant (in d dimensions). Because of the disorder average the classical return probability does not depend on $\boldsymbol{r}$. In first order, we have

$$K_1(t) = \lambda_0 R(2t/t_{\mathrm{T}})/2t. \quad (14)$$

Combining this with the coth function in Eq. (10) we find

$$\Omega_1^{(D)} = \lambda_0\frac{L\hbar}{2\pi}\sum_{m=-\infty}^{+\infty}\cos\left(\frac{4\pi m\phi}{\phi_0}\right) g_m(T) \quad (15)$$

with

$$g_m(T) = \int_0^\infty \mathrm{d}t \frac{R^2(t/t_{\mathrm{T}})}{t^2}\frac{\exp[-(mL)^2/(4Dt)]}{\sqrt{4\pi Dt}}. \quad (16)$$

Taking the derivative with respect to the flux, we recover the first-order interaction contribution to the persistent current, first obtained in Ref. [19] by purely diagrammatic techniques

$$I_1 = \lambda_0\frac{Le}{\pi}\sum_{m=-\infty}^{+\infty} m\sin\left(\frac{4\pi m\phi}{\phi_0}\right) g_m(T). \quad (17)$$

Semiclassically, this first-order result was already derived by Montambaux [14].

In addition, our semiclassical approach allows us to obtain the renormalization of the coupling constant [16–20] due to the higher-order diagrams of the Cooper series. Including these diagrams amounts to using the full kernel $K(t)$ in Eq. (10) instead of $K_1(t)$. Introducing the Laplace transform of $K_1(t)$

$$\hat{f}(p) = 2\lambda_0\sum_{n=0}^{n_{\mathrm{F}}}\frac{1}{pt_{\mathrm{T}}+2(2n+1)} \quad (18)$$

($n_{\mathrm{F}} = \beta E_{\mathrm{F}}/2\pi = k_{\mathrm{F}}L_{\mathrm{T}}/4$), $K(t)$ is given by the inverse Laplace transform

$$K(t) = \frac{1}{2\pi\mathrm{i}}\int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty}\mathrm{d}p\,\mathrm{e}^{+pt}\ln[1+\hat{f}(p)] \simeq \frac{2}{\lambda_0\ln(k_{\mathrm{F}}L^*)}K_1(t), \quad L^* = \min(v_{\mathrm{F}}t, L_{\mathrm{T}}/4). \quad (19)$$

The last equality is valid when $\ln k_{\mathrm{F}}L^* \gg 1$ which is certainly satisfied when $\ln k_{\mathrm{F}}l \gg 1$. Therefore, the higher-order terms merely lead to a renormalization of the coupling constant, thus reducing the predicted magnitude of the persistent current. In the

high-temperature regime ($L_T \ll L_m$) the coupling constant is renormalized to $2/\ln(k_F L_T/4)$. Introducing $L_m = v_F(mL)^2/4D$, the average length of a trajectory diffusing m times around the ring, one gets at low temperature ($L_T \gg L_m$) a replacement of $\lambda_0 \equiv 1$ by$2/\ln(k_F L_m)$.These two limits agree with results obtained diagrammatically by Eckern [20].

We note that the functional form of the temperature dependence (exponential T-damping [19]) is in line with experiments [4–6] while the amplitude of the persistent current with renormalized coupling constant is smaller than the experiments by a factor of ~ 5.

5. Diffusive two-dimensional systems

Contrary to rings, the geometry imposes no shortest length for returning paths in singly-connected systems. One, therefore, expects a different temperature dependence of the magnetic response.

Consider a two-dimensional singly-connected diffusive quantum dot. In view of the general renormalization property of diffusive systems, Eq. (19), the diagonal part of the thermodynamic potential from the entire Cooper series [Eq. (10)] can be written as

$$\Omega^{(D)} = \frac{1}{\beta}\int \mathrm{d}\boldsymbol{r}\int \mathrm{d}t \frac{1}{\ln(k_F v_F t)}\frac{t_T}{t^2}R^2\left(\frac{t}{t_T}\right)\mathscr{A}(\boldsymbol{r},t;B). \tag{20}$$

Here we have used $L^* = v_F t$ in Eq. (19) since the R^2 factor ensures that the main contribution to the integral comes from $t < t_T$. In two dimensions the conditional return probability, entering into $\mathscr{A}$, is conveniently expressed in terms of the Fourier transform [9]

$$P(\boldsymbol{r},\boldsymbol{r},t|A) = \frac{1}{4\pi^2}\int \mathrm{d}k|k|\mathrm{e}^{\mathrm{i}kA}\frac{\exp(-|k|Dt)}{1-\exp(-2|k|Dt)} \tag{21}$$

from which one obtains

$$\mathscr{A}(\boldsymbol{r},t;B) = \frac{1}{4\pi D}\frac{R(t/t_B)}{t}. \tag{22}$$

Here, we introduced the magnetic time

$$t_B = \frac{\phi_0}{4\pi BD} = \frac{L_B^2}{4\pi D}. \tag{23}$$

It is related to the square of the magnetic length L_B^2 which denotes the area enclosing one flux quantum (assuming diffusive dynamics). Note that the function R in Eq. (22) has a different origin than in Eq. (20).

Using Eq. (22) in Eq. (20) and taking the second derivative with respect to the field, we find for the susceptibility

$$\frac{\chi^{(D)}}{|\chi_L|} = -\frac{12}{\pi}(k_F l)\int_{\tau_{el}}^{\infty}\frac{\mathrm{d}t}{t\ln(k_F v_F t)}R^2\left(\frac{t}{t_T}\right)R''\left(\frac{t}{t_B}\right), \tag{24}$$

where R'' is thesecond derivative of R. The susceptibility is normalized to the two-dimensional diamagnetic Landau susceptibility$\chi_L = -e^2/(12\pi mc^2)$.

In the above time integral the elastic scattering time $\tau_{el} = l/v_F$ enters as a lower bound. This cutoff must be introduced since for backscattered paths withtimes shorter than τ_{el} the diffusion approximation, Eq. (21), no longer holds [23]. On the other hand, Eq. (24) holds true only as long as the upper cutoff time $t^* \equiv \min(t_T, t_B)$ is smaller than the Thouless time $t_c = L^2/D$ (with L being the system size). For times larger than t_c the dynamics begins to behave ergodically, and the two-dimensional diffusion approximation is no longer valid. Assuming $t^* < t_c$, Eq. (24) can be approximately evaluated by replacing $R(t/t_T)$ and $R''(t/t_B)$ by $R(0) = 1$ and $R''(0) = -\frac{1}{3}$, respectively, and introducing the upper cutoff t^* in the integral. The remaining integral yields for $t^* \gg \tau_{el}$

$$\int_{\tau_{el}}^{t^*}\frac{\mathrm{d}t}{t\ln(k_F v_F t)} = \ln\left\{\frac{\ln[k_F v_F \min(t_T, t_B)]}{\ln(k_F l)}\right\}. \tag{25}$$

The log–log form produced by the $1/t\ln t$ dependence results from the wide distribution of path lengths in the system – there are flux-enclosing paths with lengths ranging from about $v_F\tau_{el}$ up to $v_F t^*$. In contrast, in the ring geometry discussed in the previous section the temperature dependence is exponential because the minimum length of flux-enclosing trajectories is the circumference.

The averaged susceptibility of a diffusive two-dimensional structure then reads

$$\frac{\chi^{(D)}}{|\chi_L|} \simeq \frac{4}{\pi}(k_F l)\ln\left\{\frac{\ln[k_F v_F \min(t_T, t_B)]}{\ln(k_F l)}\right\}. \tag{26}$$

One thus finds a log–log temperature dependence for $t_T < t_B$ and a log–log B dependence for $t_T > t_B$.With

regard to magnitude, the magnetic response of diffusive systems is paramagnetic and enhanced by a factor $k_F l$ compared to the clean Landau susceptibility χ_L.

Eq. (26) agrees with results from Aslamazov and Larkin [16], Altshuler et al. [17, 18] obtained with quantum diagrammatic perturbation theory.The equivalence between the semiclassical and quantum approaches to diffusive systems may be traced back to the fact that the "quantum"diagrammatic perturbation theory relies on the use of the small parameter $1/k_F l$ which can be viewed as a semiclassical approximation.

6. Conclusions

To conclude, we developed a semiclassical approach to evaluate the interaction contribution of the grand potential in a high-density perturbative expansion. We showed that the averaged quantum magnetic response can be expressed in terms of an operator containing the classical probability for particles to return. As an application we computed the orbital magnetic response of diffusive rings and two-dimensional quantum dots arising from the combined effects of disorder and interaction.

Acknowledgements

RAJ and KR acknowledge support from the French–German program PROCOPE. The Division de Physique Théorique is "Unité de recherche des Universités Paris 11 et Paris 6 associée auC.N.R.S.".

References

[1] A.L. Efros, M. Pollak (Eds.), Electron–Electron Interactions in Disordered systems, North-Holland, Amsterdam, 1985.
[2] U. Sivan, R. Berkovits, Y. Aloni, O. Prus, A. Auerbach, G. Ben-Yoseph, Phys. Rev. Lett. 77 (1996) 1123.
[3] O. Agam, N.S. Wingreen, B.L. Altshuler, D.C. Ralph, M. Tinkham, Phys. Rev. Lett. 78 (1997) 1956.
[4] L.P. Lévy, G. Dolan, J. Dunsmuir, H. Bouchiat, Phys. Rev. Lett. 64 (1990) 2074.
[5] V. Chandrasekhar, R.A. Webb, M.J. Brady, M.B. Ketchen, W.J. Gallagher, A. Kleinsasser, Phys. Rev. Lett. 67 (1991) 3578.
[6] P. Mohanty, E.M.Q. Jariwala, M.B. Ketchen, R.A. Webb, in: K. Fujikawa, Y.A. Ono (Eds.), Quantum Coherence and Decoherence, Elsevier, Amsterdam, 1996.
[7] For recent reviews see, e.g. U. Eckern, P. Schwab, Adv. Phys. 44 (1995) 387, and Ref. [8].
[8] K. Efetov, Supersymmetry in Disorder and Chaos, Cambridge University Press, Cambridge, 1996.
[9] N. Argaman, Y. Imry, U. Smilansky, Phys. Rev. B 47 (1993) 4440.
[10] F. von Oppen, E.K. Riedel, Phys. Rev. B 48 (1993) 9170.
[11] F. von Oppen, Phys. Rev. B 50 (1994) 17151.
[12] D. Ullmo, K. Richter, R.A. Jalabert, Phys. Rev. Lett. 74 (1995) 383.
[13] K. Richter, D. Ullmo, R.A. Jalabert, Phys. Rep. 276 (1996) 1.
[14] G. Montambaux, J. Physique 6 (1996) 1.
[15] D. Ullmo, H.U. Baranger, K. Richter, F. von Oppen, R.A. Jalabert, Report No. cond-mat/9708092 and unpublished.
[16] L.G. Aslamazov, A.I. Larkin, Sov. Phys. JETP 40 (1975) 321.
[17] B.L Altshuler, A.G. Aronov, A.Yu. Zyuzin, Sov. Phys. JETP 57 (1983) 889.
[18] For a review see B.L. Altshuler, A.G. Aronov in Ref.[1].
[19] V. Ambegaokar, U. Eckern, Phys. Rev. Lett. 65 (1990) 381.
[20] U. Eckern, Z. Phys. B 42 (1991) 389.
[21] A.A. Abrikosov, L.P. Gorkov, I.E. Dzyaloshinski, Methods of Quantum Field Theory in Statistical Physics, Prentice-Hall, Englewood Cliffs, NJ, 1963.
[22] M.-J. Giannoni, A. Voros, J. Zinn-Justin (Eds.), Chaos and Quantum Physics, North-Holland, New York, 1991.
[23] Shorter paths with $t < \tau_{el}$ arise from higher-order interaction events and contribute to the clean bulk magnetic response which however is negligible compared to the disorder induced interaction contribution [15].