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Semiclassical approach to orbital magnetism of interacting diffusive quantum systems

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Abstract

We study interaction effects on the orbital magnetism of diffusive mesoscopic quantum systems. By combining many-body perturbation theory with semiclassical techniques, we show that the interaction contribution to the ensemble-averaged quantum thermodynamic potential can be reduced to an essentially classical operator. We compute the magnetic response of disordered rings and dots for diffusive classical dynamics. Our semiclassical approach reproduces the results of previous diagrammatic quantum calculations. © 1997 Elsevier Science B.V. All rights reserved.

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1. Introduction

The interplay of disorder and interactions in mesoscopic systems has attracted considerable attention [1]. Interaction effects on transport through small quantum dots [2, 3] as well as on thermodynamic properties like persistent currents and orbital magnetism are of present interest. In the latter case, the unexpectedly large measured persistent current of small metal rings [4–6] pointed towards the importance of such interaction effects and motivated a large number of theoretical approaches [7, 8].

For the description of thermodynamic quantities, semiclassical expansions have proven particularly useful, both within the independent-particle model [9–13] and for interaction effects [14, 15]. These studies established a close relation between the classical dynamics and the quantum-mechanical magnetic response. In particular, studies of ballistic systems showed that the quantum thermodynamic properties are sensitive to whether the classical dynamics is regular or chaotic [10–13, 15].

In this paper we apply these semiclassical techniques to the orbital magnetism of interacting sys-

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tems whose non-interacting classical dynamics is *diffusive*. Specifically, we present semiclassical derivations of the interaction contributions to the persistent current of metal rings and to the suscepti-

of states and the bookkeeping index $\lambda_0 = 1$ identifies the order of perturbation. For the local interaction, direct and exchange term are equivalent up to a factor of (-2) due to the spin sums and the different num-

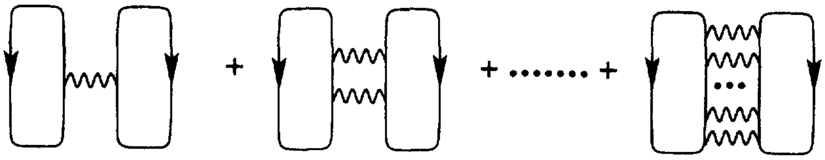

Fig. 1. Leading Cooper-channel diagrams for the interaction contribution to thethermodynamic potential.

i.e., that the cyclotron radius $R_c \gg \min\{l, L\}$ (with l the elastic mean free path and L the system size).

Semiclassically, the retarded Green's function is represented as a sum of contributions $G^{R;j}_{r,r'}$ over all classical paths j from r to r' [22],

$$G^{R}_{r,r'}(E) \simeq \sum_{j:r\to r'} D_j \, e^{iS_j/\hbar - i\pi\nu_j/2}. \qquad (5)$$

Here $S_j = \int_r^{r'} \boldsymbol{p}\cdot d\boldsymbol{r}$ is the classical action of trajectory j. The prefactor D_j includes the classical phase-space density $[D_j = (1/\sqrt{2\pi(i\hbar)^3 \dot{x}\dot{x}'})|\partial^2 S_j/\partial y \partial y'|^{1/2}$ in two dimensions]. ν_j is a Maslov index. The semiclassical approximation makes the temperature and magnetic-field dependences of the finite-temperature Green's function transparent. Employing $(\partial S_j/\partial E) = t_j$ and $(\partial S_j/\partial B) = (e/c)A_j$, where t_j and A_j are the traversal time and area, one finds

$$G^{R;j}_{r,r'}(E_F + i\varepsilon, B) \simeq G^{R;j}_{r,r'}(E_F, B=0) \times \exp[-\varepsilon t_j/\hbar] \times \exp[i2\pi B A_j/\phi_0] \qquad (6)$$

where $\phi_0 = hc/e$ is the flux quantum. Note that temperature exponentially suppresses the contributions of long paths to each Green's function.

Semiclassically, the particle–particle propagator $\Sigma_{r,r'}(\omega)$ is then represented as a sum over pairs of paths between r and r'. Off-diagonal pairs (of different paths) generally contain highly oscillatory contributions which do not survive an ensemble (disorder) average. (There can be exceptions as discussed in Ref. [15].) On the other hand, the diagonal pairing of each orbit j with its time reverse persists upon averaging since their dynamical phases $\exp[iS_j(B=0)/\hbar]$

Using Eqs. (4)–(6) in Eq. (3) and performing the Matsubara sum yields for the diagonal part of $\hat{\Sigma}$

$$\Sigma^{(D)}_{r,r'}(\omega) \simeq \frac{\hbar}{\pi N(0)} \sum_{j:r\to r'}^{L_j > \Lambda_0} |D_j|^2 \frac{R(2t_j/t_T)}{2t_j} \times \exp\left[\frac{i4\pi B A_j}{\phi_0}\right] \times \exp\left[-\frac{\omega t_j}{\hbar}\right]. \qquad (7)$$

The sum runs over all trajectories longer than the cutoff $\Lambda_0 = \lambda_F/\pi$ [corresponding to the upper bound E_F on the Matsubara sum in Eq. (3)]. The temperature dependence in Eq. (7) enters through the function $R(x) = x/\sinh(x)$ introducing the time scale

$$t_T = \frac{\hbar\beta}{\pi} \qquad (8)$$

and the related length scale $L_T = v_F t_T$, with v_F being the Fermi velocity. This semiclassical framework allows us to reduce the original quantum problem to $\Sigma^{(D)}$, which no longer exhibits variations on the quantum scale λ_F but only on classical scales. We emphasize that the representation, Eq. (7), of $\Sigma^{(D)}$ is rather general since we have not yet made any assumption about the classical dynamics of the system. In particular, it applies to both diffusive and ballistic systems. On the basis of Eq. (7), we have recently studied interaction effects in ballistic quantum dots [15]. Specifically, we show that the interaction-induced orbital magnetism scales differently for systems with regular and chaotic non-interacting classical counterparts.

Here, we focus on diffusive systems for which it is useful to relate $\Sigma^{(D)}$ to classical probabilities satisfying the diffusion equation. To th[illegible]

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between the weights $|D_j|^2$ and the classical probability $P(\boldsymbol{r},\boldsymbol{r}';t)$ to propagate from $\boldsymbol{r}$ to $\boldsymbol{r}'$ in time t.

An nth order contribution to Ω in Eq. (2) then contains expressions for the joint return probability $P(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_n,\boldsymbol{r}_1;t_1,\ldots,t_n|A)$ to visit the points $\boldsymbol{r}_i$ (with t_i being the time between $\boldsymbol{r}_i$ and $\boldsymbol{r}_{i-1}$) under the condition that the enclosed area is A. In

where $D = v_{\mathrm{F}} l/d$ is the diffusion constant (in d dimensions). Because of the disorder average the classical return probability does not depend on $\boldsymbol{r}$. In first order, we have

$$K_1(t) = \lambda_0 R(2t/t_{\mathrm{T}})/2t. \tag{14}$$

Combining this with the coth function in Eq. (10) we

[redacted]

namely ∫ dr ... dr P(r ... r ; t ... | A) =

[redacted]

high-temperature regime ($L_T \ll L_m$) the coupling constant is renormalized to $2/\ln(k_F L_T/4)$. Introducing $L_m = v_F(mL)^2/4D$, the average length of a trajectory diffusing m times around the ring, one gets at low temperature ($L_T \gg L_m$) a replacement of $\lambda_0 \equiv 1$ by$2/\ln(k_F L_m)$.These two limits agree with results obtained diagrammatically by Eckern [20].

We note that the functional form of the temperature dependence (exponential T-damping [19]) is in line with experiments [4–6] while the amplitude of the

(assuming diffusive dynamics). Note that the function R in Eq. (22) has a different origin than in Eq. (20).

Using Eq. (22) in Eq. (20) and taking the second derivative with respect to the field, we find for the susceptibility

$$\frac{\chi^{(D)}}{|\chi_L|} = -\frac{12}{\pi}(k_F l) \int_{\tau_{el}}^{\infty} \frac{dt}{t \ln(k_F v_F t)} R^2\left(\frac{t}{t_T}\right) R''\left(\frac{t}{t_B}\right), \quad (24)$$

where R'' is thesecond derivative of R. The regard to magnitude, the magnetic response of diffusive systems is paramagnetic and enhanced by a factor $k_F l$ compared to the clean Landau susceptibility χ_L.

Eq. (26) agrees with results from Aslamazov and Larkin [16], Altshuler et al. [17, 18] obtained with quantum diagrammatic perturbation theory.The equivalence between the semiclassical and quantum approaches to diffusive systems may be traced back to the fact that the "quantum"diagrammatic perturbation theory relies on the use of the small parameter $1/k_F l$ which can be viewed as a semiclassical approximation.

6. Conclusions

To conclude, we developed a semiclassical approach to evaluate the interaction contribution of the

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