PRAMANA — journal of physics | © Indian Academy of Sciences | Vol. xx, No. x
xxxx xxxx
pp. 1–10

Kondo effect and mesoscopic fluctuations

DENIS ULLMO[a,*], SÉBASTIEN BURDIN[c,†], DONG E. LIU[b,‡], HAROLD U. BARANGER[b,§]
[a]Univ. Paris Sud, CNRS, LPTMS, 91405 Orsay Cedex, France
[b]Department of Physics, Duke University, Box 90305, Durham, NC 27708-0305, USA
[c]Condensed Matter Theory Group, LOMA, UMR 5798, Université de Bordeaux I, 33405 Talence, France

Abstract. Two important themes in nanoscale physics in the last two decades are correlations between electrons and mesoscopic fluctuations. Here we review our recent work on the intersection of these two themes. The setting is the Kondo effect, a paradigmatic example of correlated electron physics, in a nanoscale system with mesoscopic fluctuations; in particular, we consider a small quantum dot coupled to a finite reservoir (which itself may be a large quantum dot). We discuss three aspects of this problem. First, in the high temperature regime, we argue that a Kondo temperature $T_{\rm K}$ which takes into account the mesoscopic fluctuations is a relevant concept: for instance, physical properties are universal functions of $T/T_{\rm K}$. Second, when the temperature is much less than the mean level spacing due to confinement, we characterize a natural cross-over from weak to strong coupling. This strong coupling regime is itself characterized by well-defined single particle levels, as one can see from a Nozières Fermi-liquid theory argument. Finally, using a mean-field technique, we connect the mesoscopic fluctuations of the quasi-particles in the weak coupling regime to those at strong coupling.

Keywords. Kondo, quantum dots, finite size effects, mesoscopic

PACS Nos. 75.20.H,73.21.La,05.45.Ac

1. Introduction

The term “Kondo effect” refers to the physics of a local quantum system with an internal degree of freedom—often referred to as a “quantum impurity”—interacting with a gas of otherwise non-interacting electrons. It represents one of the simplest models in condensed matter physics for which *correlations* play a central role [1]. On the other hand, “mesoscopic fluctuations” refers to the variation of physical properties when one considers a quantum coherent region: the differing interference contributions for nominally the same system lead to sample-to-sample variation termed fluctuations [2]. While each of these topics has been intensively investigated individually, their intersection—the interplay of mesoscopic fluctuations and correlated electrons—has not. Yet as strong

*denis.ullmo@u-psud.fr
†s.burdin@cpmoh.u-bordeaux1.fr
‡dl35@phy.duke.edu
§baranger@phy.duke.edu

correlations, such as the Kondo effect, continue to be investigated in nanoscale systems where mesoscopic fluctuations are ubiquitous, study of the intersection is very natural. We have carried out investigations of several aspects of mesoscopic Kondo physics in the last five years [3–7], and we summarize and integrate that work in this article.

In the simplest version of the Kondo problem—the *s-d* model (also called the Kondo model)—the impurity is just treated as a quantum spin one half interacting locally with the electron gas [1]. The corresponding Hamiltonian then reads

$$H_{\rm K} = \sum_{\alpha\sigma} \epsilon_\alpha \hat{c}^\dagger_{\alpha\sigma} \hat{c}_{\alpha\sigma} + H_{\rm int} \ , \tag{1}$$

where $\hat{c}^\dagger_{\alpha\sigma}$ creates a particle with energy ϵ_α, spin σ and wave-function $\varphi_\alpha(\mathbf{r})$. The (anti-ferromagnetic) interaction with the impurity is

$$H_{\rm int} = \frac{J_0}{\hbar^2} \, \mathbf{S} \cdot \mathbf{s}(\mathbf{0}) \tag{2}$$

with $J_0 > 0$ the coupling strength, $\mathbf{S} = (S_x, S_y, S_z)$ a quantum spin $1/2$ operator ($\hbar^{-1} S_i$ is half of the Pauli matrix σ_i), $\mathbf{s}(0) = \frac{\hbar}{2} \hat{\Psi}^\dagger_\sigma(0) \sigma_{\sigma\sigma'} \hat{\Psi}_\sigma(0)$ the spin density of the electron gas at the impurity position $\mathbf{r} = 0$, and $\hat{\Psi}^\dagger_\sigma(0) = \sum_\alpha \varphi_\alpha(0) \hat{c}^\dagger_\alpha$.

Originally, physical realizations of the Kondo Hamiltonian were actual impurities (e.g. *Fe*) in a bulk metal (e.g. *Au*). The wave-functions φ_α could then be taken as plane waves, and one could assume a constant spacing Δ between the ϵ_α, so that the electron gas could be characterized by only two quantities: the local density of states $\nu_0 = (\mathcal{A}\Delta)^{-1}$ ($\mathcal{A}$ is the volume of the sample) and the bandwidth D_0 of the spectrum.

What gives the Kondo problem its particular place in condensed matter physics is that it is the simplest problem for which the physics is dominated by renormalization effects. Indeed, assuming the dimensionless constant $J_0\nu_0 \ll 1$, it can be shown using one-loop renormalization group analysis [8], or equivalent earlier approaches such as Anderson poor man's scaling [9] or Abrikosov re-summation of parquet diagrams [10], that the low energy physics remains unchanged by the simultaneous change of J_0 and D_0 to new values $J_{\rm eff}$ and $\mathrm{D}_{\rm eff}$ provided they are related by

$$J_{\rm eff}(\mathrm{D}_{\rm eff}) = \frac{J_0}{1 - J_0\nu_0 \ln(\mathrm{D}_0/\mathrm{D}_{\rm eff})} \ . \tag{3}$$

The renormalization procedure should naturally be stopped when $\mathrm{D}_{\rm eff}$ becomes of order the temperature T of the system. Equation (3) defines an energy scale, the Kondo temperature

$$T_{\rm K} = \mathrm{D}_0 \exp(-1/J_0\nu_0) \ , \tag{4}$$

which specifies the crossover between the weakly and strongly interacting regimes. For $T \gg T_{\rm K}$, the impurity is effectively weakly coupled to the electron gas, and the properties of the system can be computed within a perturbative approach provided the renormalized interaction $J_{\rm eff}(T)$ is used. The regime $T \ll T_{\rm K}$ is characterized by an effectively very strong interaction(despite the bare coupling value $J_0\nu_0$ being small) in such a way that the spin of the impurity is almost completely screened by the electron gas. Perturbative renormalization analysis [and thus(3)] can not be applied in this regime, but a rather complete description of clean bulk systems has been obtained by a variety of approaches, including numerical renormalization group [11], Bethe ansatz techniques[12, 13], and the Nozières Fermi liquid description [14].

One important consequence of the scaling law (3) is that physical quantities can be described by *universal* functions. This can be understood by a simple counting of the number of parameters defining the *s-d* model in the bulk. As an illustration, we will use the local magnetic susceptibility, defined by

$$\chi_{\rm loc} \equiv \int_0^\beta d\tau \langle S_z(\tau) S_z(0) \rangle \ , \tag{5}$$

which is the variation of the impurity spin magnetization upon applying a field only to the impurity. The electron gas is characterized by its local density of states ν_0 and its bandwidth D_0, and the impurity by the coupling constant J_0. Therefore, for a given temperature T, $\chi_{\rm loc}$, or indeed any physical quantity, can depend only on these four parameters. Furthermore, only two dimensionless parameters can be constructed from them, the ratio T/D_0 and the product $J_0\nu_0$. However, because of the scaling law (3), these two parameters turn out to be redundant. A dimensionless quantity can therefore be expressed as a function of a *single* parameter, which is usually chosen to be $T/T_{\rm K}$. Thus we have, for instance,

$$T\chi_{\rm loc}(T) = f_\chi(T/T_{\rm K}) \ , \tag{6}$$

where $f_\chi(x)$ is a universal function which has been computed by Wilson [11] using his numerical renormalization group approach.

We see that the universal character of Kondo physics is a direct consequence of the fact that the local density of states is flat and featureless on an energy scale of the order of $\max(T_{\rm K}, T)$. There are many situations, however, for which the variation in energy of the local density of states $\nu_{\rm loc}(\mathbf{r};\epsilon)$ is significantly more complex, and it is natural to ask in what way this modifies the description given above for the bulk flat-band case. One type of non-flatness is a continuous density of states in which the variation sets in for energies above some scale, say $\delta_{\rm fl}$. In this case, deviations from universal Kondo behavior are expected either in the regime $\delta_{\rm fl} < T_{\rm K}$ for any T or in the regime $\delta_{\rm fl} > T_{\rm K}$ when $T > \delta_{\rm fl}$. Other relevant examples of deviations emerge in mesoscopic systems from a discrete (and therefore non-flat) density of states, and it is to these that we now turn.

One circumstance in which such a non-trivial density of states occurs naturally is in the context of quantum dots. Indeed, as was pointed out both by Glazman and Raikh [15] and by Ng and Lee [16], a quantum dot containing an odd number of electrons and which is sufficiently small so that the spacing between its levels is much larger than T can be described by an Anderson impurity model; the role of the electron gas is played by the leads to which the dot is weakly coupled. In the deep Coulomb blockade regime particle number fluctuations are suppressed, and a Schrieffer-Wolff transformation shows [1] that such a description is essentially equivalent to a Kondo impurity. In the low temperature regime $T \ll T_{\rm K}$, a correlated state is formed which mixes the wavefunctions of the quantum dot and both leads, leading to a large conductance ($\simeq e^2/h$) despite the Coulomb blockade. These predictions were confirmed experimentally a decade later [17], and subsequently Kondo physics has been seen in a variety of nanoscale systems [18, 19], some of which have even reached the unitarity limit.

In quantum dot Kondo physics, one is very naturally lead to systems in which a small quantum dot playing the role of a quantum impurity is connected to a larger mesoscopic object (itself not a quantum impurity) in which finite size and interference effects are important. In other words, the context of Kondo physics in quantum dots leads one to consider situations where the “quantum impurity” is connected to an electron gas displaying finite size and mesoscopic fluctuation effects. As a consequence, for each such mesoscopic electron reservoir, two new energy scales enter into the description of the Kondo problem: the corresponding mean level spacing Δ and the Thouless energy $E_{\rm Th}$. The former is the scale at which the discreetness of the density of states becomes relevant, and the latter is the scale below which mesoscopic fluctuations set in. Our goal in this short review is to provide a survey of some of our own results [3–7] describing how Kondo physics is modified by the presence of these energy scales.

2. The $T \gg \Delta$ regime

Let us consider, to start with, a situation where the mean level spacing Δ in the reservoirs is still much smaller than the other relevant energy scales (in particular the temperature) but for which the Thouless energy $E_{\rm Th}$ of the reservoir is significant [3, 4]. Assuming ballistic motion, $E_{\rm Th}$ is the inverse of the time of flight across the system (up to a factor $\hbar$). It characterizes the range of energy below which mesoscopic fluctuations are present (at all scales in this range), in particular in the local density of states. Since below $E_{\rm Th}$ the reservoir's electronic density of states is not flat and featureless, one may wonder whether a Kondo temperature (perhaps fluctuating) can be defined, and if physical quantities remain universal functions of the ratio $T/T_{\rm K}$.

To fix the ideas, let us consider the *s*-*d* Hamiltonian (1) with a local density of states at the impurity site $\nu(\mathbf{r}=\mathbf{0};\epsilon) \stackrel{\rm def}{=} \sum_{\mathbf{i}} |\varphi_{\mathbf{i}}(\mathbf{0})|^{\mathbf{2}} \delta(\epsilon-\epsilon_{\mathbf{i}})$. In the semiclassical regime, $\nu(\mathbf{r}\!=\!\mathbf{0};\epsilon)$ can be written as the sum

$$\nu(\epsilon) = \nu_0 + \nu_{\rm fl}(\epsilon) \tag{7}$$

where ν_0 is the bulk-like contribution (one should of course include here either a realistic band dispersion relation or a cutoff at $\rm D_0$ to account for the finite bandwidth), and the fluctuating term $\nu_{\rm fl}(\epsilon)$ is a quantum correction associated with interference effects.

Let us denote by $T_{\rm K}^0$ the Kondo temperature of the associated bulk system for which $\nu(\epsilon)$ is replaced by ν_0. For $T \gg T_{\rm K}^0$ it is possible to use a perturbative renormalization group approach in the same way as in the bulk, but including the mesoscopic fluctuations of the density of states. Following [20], this gives in the one-loop approximation

$$J_{\rm eff}({\rm D_{eff}}) = \frac{J_0}{1 - J_0 \int_{\rm D_{eff}}^{\rm D_0} (d\omega/\omega)\, \nu_\beta(\omega)} \ , \tag{8}$$

with

$$\nu_\beta(\omega) \equiv \frac{\omega}{\pi} \int_{-\infty}^{\infty} d\epsilon \frac{\nu_{\rm loc}(\epsilon)}{\omega^2+\epsilon^2} \tag{9}$$

the temperature smoothed density of states (note that the renormalization up to two loop order was given in Ref. [20]).

There are two different ways to use the above renormalization group equation. First, for some physical systems, the fluctuations of the local density of states may be very significant, yielding even larger variation of the Kondo properties because of the exponential dependence in (4). In this case, one is mainly interested in the fluctuations of the Kondo temperature (which is now a functional of the local density of states) defined as the *energy scale* separating the weak and strong coupling regimes. One can then use the same approach as in the bulk flat-band case and define $T_{\rm K}[\nu_\beta]$ as the temperature at which the one-loop effective interaction diverges, giving the implicit equation

$$J_0 \int_{T_{\rm K}^*[\nu_\beta]}^{\rm D_0} \frac{d\omega}{\omega} \nu_\beta(\omega) = 1 \ . \tag{10}$$

Examples of systems for which the fluctuations of ν_β are large enough so that one is mainly interested in the fluctuations of the scale $T_{\rm K}^*$ defined by (10) include, for instance, the case of "real" (chemical) impurities in a geometry such than one dimension is not much larger than the Fermi wavelength. This may be either a quantum point contact in a two dimensional electron gas [20] or a thin three dimensional film [21]. There is a significant chance that the impurity is located in a place at which the Friedel oscillations are large (i.e. the term in $\nu_{\rm fl}$ associated with the trajectory bouncing back from the boundary directly to its starting point). In this case, impurities located at peaks of the Friedel oscillations have a significantly larger Kondo temperature than those at the troughs. Disordered metals near or beyond the localization transition are another system with large fluctuations of the local density of states, and can lead in particular to a finite probability of having a "free moment" for which $T_{\rm K}^*[\nu_\beta]$ corresponding to certain spatial locations is actually zero [22, 23].

In contrast to these cases, in a typical ballistic or disordered-metallic mesoscopic system with all dimensions much larger than the Fermi wavelength, the fluctuating part of the local density of states $\nu_{\rm fl}$ is a quantum correction to the secular term ν_0 and thus parametrically smaller. Consequently, the fluctuations of $T_{\rm K}^*[\nu_\beta(\omega)]$ defined by (10) are not large compared to $T_{\rm K}^0$, which if it is taken only as an energy scale is somewhat meaningless.

In the flat-band bulk case, however, $T_{\rm K}$ has a precise meaning beyond being an energy scale: it is the parameter entering into universal functions such as f_χ in (6). $T_{\rm K}$ is thus directly and quantitatively related to physical observables—its fluctuations need not be large to be relevant. This suggests asking whether the mesoscopic fluctuations of physical observables such as the local susceptibility $\chi_{\rm loc}$ are described by the universal form

$$T\chi_{\rm loc}(T) = f_\chi(T/T_{\rm K}[\nu_\beta]) \ , \tag{11}$$

with all the mesoscopic fluctuations encoded in the realization and position dependent Kondo temperature $T_{\rm K}[\nu_\beta]$).

In the high temperature regime $T \geq T_{\rm K}^0$ such universality follows from a perturbative renormalization argument (although not so straightforwardly, c.f. the discussion in Refs. [3, 24]). Furthermore, comparison with quantum Monte Carlo results shows that predictions obtained in this way are quantitatively very accurate [3, 4]. More surprisingly, such an approach remains valid up to (and actually somewhat below) $T_{\rm K}^0$: in the high temperature regime, then, mesoscopic fluctuations can be included rather easily into Kondo physics. As perturbative renormalization group is not valid below $T_{\rm K}$, Eq. (11) is not expected to apply in the low temperature regime $T \ll T_{\rm K}^0$. This is indeed confirmed by quantum Monte Carlo cacuulations [3]: the low temperature regime of the mesoscopic Kondo problem is therefore richer and more complex.

3. The $T \ll \Delta$ regime

Let us now consider the temperature regime $T \ll \Delta$ where the discreetness of the spectrum has to be taken into account [5, 6]. In this temperature range, the physical properties of the system will be dominated by its many-body ground state and first few excitations. For illustration, we consider a small quantum dot (the quantum impurity) connected to a large mesoscopic quantum dot reservoir, as shown in Fig. 1. In the case of an odd number of electrons in the reservoir dot, a sketch of the low energy excitations is shown in Fig. 1. Without reproducing the details of the analysis given in [5, 6] (where the even N case as well as other Kondo systems are also discussed), we shall make a few comments on this figure and on the way it has been constructed.

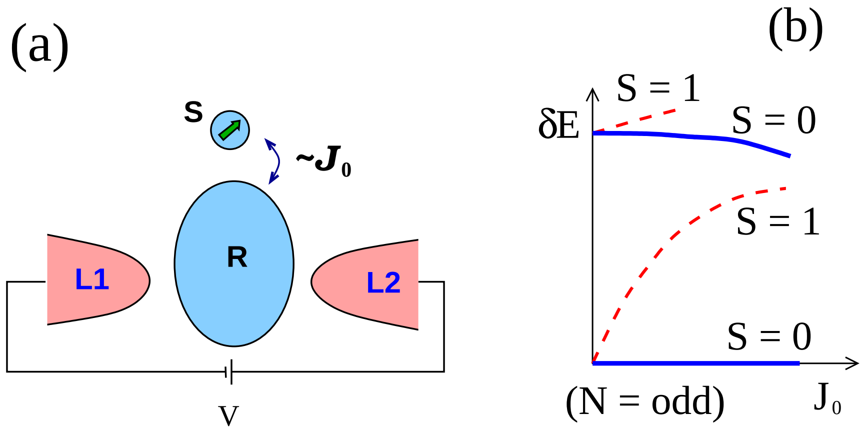

Figure 1. (a) A double dot system, coupled very weakly to leads ($L1$ and $L2$). In the Coulomb blockade regime, the small dot behaves as a spin S that is coupled to a finite reservoir R provided by the large dot. The leads are used to measure the excitation spectrum of the system. (b) Sketch of the ground state and first excitations of the mesoscopic Kondo problem as a function of coupling strength $J_0\nu_0$.

First, we stress that the total spin of the ground state can be rigorously shown to be zero for all for positive values of J [5, 6]. The rest of the diagram can be obtained by analyzing the two limiting regimes $\Delta \gg T_{\rm K}^0$ and $\Delta \ll T_{\rm K}^0$ and assuming a smooth interpolation between them. ($T_{\rm K}^0$ is, as before, the Kondo temperature of the analogous flat-band bulk system.) The small J behavior follows from a perturbative treatment of the (renormalized) coupling $J_{\rm eff}$ between the two dots. The ground state and first excitation are the singlet and triplet states of two weakly interacting spins (all orbitals other than the last one (singly occupied) being frozen). The second excitation corresponds to the promotion of a particle to a higher orbital, and therefore involves the energy spacing Δ_2 between these two orbitals. Because of mesoscopic fluctuations, Δ_2 fluctuates around the mean level spacing Δ if one applies a weak magnetic field or slightly changes the shape of the reservoir dot.

The strong-coupling limit $\Delta \ll T_{\rm K}^0$ can, on the other hand, be addressed in the spirit of Nozières' Fermi-liquid picture: in the strong interaction regime, the impurity spin is locked in a singlet with an electron gas quasiparticle, the other N–1 quasiparticles forming an essentially non-interacting Fermi gas [1, 14]. The ground state and two first excitations sketched in Fig. 1 can therefore be understood as arising from the corresponding Fermi gas: doubly occupied orbitals for the ground state, and promotion of a particle to the first unoccupied orbital for the excitations (involving therefore the corresponding level spacing $\Delta_3 \neq \Delta_2$). The degeneracy between the singlet and triplet excitations is lifted by the residual interaction generated at the impurity site [14] by virtual breaking of the Kondo singlet.

In order to connect the levels in the weak coupling regime to those at strong coupling, we note that the order of the spins of the states is the same in both regimes and so simply draw interpolating lines between them in Fig. 1.

Both the weak-coupling and strong-coupling limiting regimes are Fermi gases—by construction for weak interactions and because of the Nozières Fermi liquid picture [14] for strong interactions. For a finite size fully coherent system these Fermi gases will both display mesoscopic fluctuations of the one particle energies and wavefunctions—in the case above, for instance, both Δ_2 and Δ_3 are expected to show mesoscopic fluctuations around Δ when some external parameter (magnetic field, gate voltage, etc.) is varied. We would like to address now the question of characterizing these fluctuations, and in particular the correlations between the Nozières Fermi gas and the weak interaction one.

4. Slave boson/fermion mean field approach

A complete solution of this problem would presumably involve developing a Fermi liquid theory "à la Nozières" [14] taking properly into account the mesoscopic fluctuations. A first step in this direction is to use a mean field treatment based on the slave boson/fermion technique [1], which can be done as follows.

Starting from the *s*-*d* model Eq. (1), one can introduce Abrikosov fermions [10] as a representation of the spin $1/2$. The correspondence between the auxiliary fermions $f_\sigma^\dagger$ with spin $\sigma = \uparrow, \downarrow$ and the initial quantum spin operators is

$$S^z \equiv [f_\uparrow^\dagger f_\uparrow - f_\downarrow^\dagger f_\downarrow]/2 \; , \quad S^+ \equiv f_\uparrow^\dagger f_\downarrow \; , \quad S^- \equiv f_\downarrow^\dagger f_\uparrow \; , \tag{12}$$

with the constraint

$$f_\uparrow^\dagger f_\uparrow + f_\downarrow^\dagger f_\downarrow = 1 \; . \tag{13}$$

The Kondo interaction can then be rewritten as

$$H_{\rm int} = \frac{J_0}{2} \sum_{\sigma\sigma'} f_\sigma^\dagger f_{\sigma'} c_{0\sigma'}^\dagger c_{0\sigma} - \frac{J_0}{4} \sum_\sigma c_{0\sigma}^\dagger c_{0\sigma} \; . \tag{14}$$

This fermionic representation is exact. The mean field approximation consists in replacing the quartic part of the Kondo term by an effective quadratic term

$$\sum_{\sigma\sigma'} f_\sigma^\dagger f_{\sigma'} c_{0\sigma'}^\dagger c_{0\sigma} \mapsto \sum_{\sigma\sigma'} \left[f_\sigma^\dagger c_{0\sigma} \langle c_{0\sigma'}^\dagger f_{\sigma'} \rangle + c_{0\sigma}^\dagger f_\sigma \langle f_{\sigma'}^\dagger c_{0\sigma'} \rangle \right] \; , \tag{15}$$

in which the mean values $\langle \cdots \rangle$ are computed self-consistently. The constraint (13) is imposed through the introduction of the Lagrange multiplyer ϵ_0:

$$H_{\rm int} \mapsto H_{\rm int} - \epsilon_0 \sum_\sigma f_\sigma^\dagger f_\sigma \; . \tag{16}$$

In the mean field approximation, the Kondo problem is therefore transformed into an effective resonant level model, which, for a given spin component, can be written as

$$H_{\rm MF} = \sum_i \left[\epsilon_i c_i^\dagger c_i + v^* \varphi_i^*(0) f^\dagger c_i + v \varphi_i(0) c_i^\dagger f^\dagger \right] - \epsilon_0 f^\dagger f \tag{17}$$

and for which the parameters v and ϵ_0 are fixed by the self-consistent equations

$$v = J_0 \sum_i \varphi_i^*(0) \langle f^\dagger c_i \rangle \; , \qquad \langle f^\dagger f \rangle = 1/2 \; . \tag{18}$$

This approach does not describe correctly the intermediate regime $\Delta, T \simeq T_{\rm K}$, and in particular predicts a phase transition at $T_{\rm K}$ instead of the observed crossover. The strong interaction / low temperature regime $\Delta, T \ll T_{\rm K}$ that we are interested in is, on the other hand, very well described.

5. Spectral fluctuations of the chaotic resonant level model

Understanding the fluctuations of the Kondo problem within the mean field approach amounts, therefore, to understanding those of the resonant level model with fluctuating parameters. These parameters are ϵ_0—the energy of the resonant level—and the coupling strength v. Let us furthermore denote the mean local density of states by $\rho \equiv < |\phi_i(0)|^2 > /\Delta$ (for a billiard of area $\mathcal{A}$, $< |\phi_i(0)|^2 > \simeq 1/\mathcal{A}$). From the physics of the Kondo problem it is expected (and found in practice) that ϵ_0 is essentially fixed at the Fermi energy and that the width $\Gamma = \pi |v|^2 \rho$ of the resonance can be identified with the Kondo temperature (up to a constant factor), the fluctuation properties of which have been addressed within the mean field approach in [25]. In the limit $T_{\rm K}^0 \simeq \Gamma \gg \Delta$ that we now consider, the fluctuations of ϵ_0 and Γ take place on a scale which is parametrically small compared to $T_{\rm K}^0$. It is thus reasonable to assume that most of the fluctuations will be given by those of the resonant level model, with the fluctuation of ϵ_0 and Γ adding only small corrections.

We therefore consider the spectral fluctuations of the resonant level model, and furthermore limit the scope to chaotic motion within the reservoir dot. Random matrix theory provides a good model of the energy levels and wavefunctions in this situation [26, 27]: we use the Gaussian orthogonal ensemble (GOE, $\beta = 1$) for time reversal symmetric systems and the Gaussian unitary ensemble (GUE, $\beta = 2$) for non-symmetric systems [26, 28]. The joint distribution function of the unperturbed reservoir dot energy levels is therefore given by [28]

$$P_\beta(\epsilon_1, \epsilon_2, \cdots, \epsilon_N) \propto \prod_{i>j} |\epsilon_i - \epsilon_j|^\beta \exp\left(-\frac{1}{4v^2} \sum_i \epsilon_i^2\right) \tag{19}$$

and the corresponding distribution of values of the wavefunction at the impurity site is the Porter-Thomas distribution

$$P_\beta(x = \mathcal{A}|\phi_j(0)|^2) = \frac{1}{(2\pi x)^{1-\beta/2}} \exp\left(-\frac{\beta}{2} x\right) \ . \tag{20}$$

To connect the mesoscopic fluctuations of the levels before and after the resonant level is added, we would like, for instance, the joint distribution of both sets of levels. For the resonant level model, the perturbed energy levels $\{\lambda_\alpha\}_{\alpha=0}^N$ are related to the unperturbed levels $\{\epsilon_i\}_{i=0}^N$ through

$$\sum_{i>0} \frac{|\phi_i(0)|^2}{\lambda_\alpha - \epsilon_i} = \frac{\lambda_\alpha - \epsilon_0}{|v|^2} \tag{21}$$

(recall that ϵ_0 is the bare position of the resonance). From this, the joint distribution function of the $\{\epsilon_i\}$ and $\{\lambda_\alpha\}$ can be computed [7]:

$$P\left(\{\epsilon_i\}, \{\lambda_\alpha\}\right) \propto \frac{\prod_{\alpha \geq 0} (\lambda_\alpha - \epsilon_0)}{\prod_{i \geq 1} (\epsilon_i - \epsilon_0)^{1-\beta/2} \prod_{i \geq 1} (\epsilon_0 - \epsilon_i)^\beta} \times \frac{\prod_{i>j} (\epsilon_i - \epsilon_j) \prod_{\alpha > \gamma} (\lambda_\alpha - \lambda_\gamma)}{\prod_{i\alpha} |\epsilon_i - \lambda_\alpha|^{1-\beta/2}}$$
$$\times \exp\left[-\frac{N\beta}{4V^2} \left(\sum_{\alpha>0} \lambda_\alpha^2 - \sum_{i>0} \epsilon_i^2\right)\right] \quad , \tag{22}$$

with in addition the constraint $\sum_{i>0} \epsilon_i = \sum_{\alpha>0} \lambda_\alpha$.

It is not completely straightforward, however, to deduce from this joint distribution useful correlation functions involving a few levels, from which we could get an intuitive sense of how the fluctuations of the unperturbed system affect those of the perturbed one. Another possible route is, in the spirit of the Wigner surmise for the nearest neighbor distribution of the classic random matrix ensemble, to start from a simple toy model, in which a good part of the information is ignored but on the other hand the expressions obtained are tractable and contain enough of the relevant information to provide a good first approximation.

To construct such a toy model, first note that Eq. (21) imposes that there is one and only one level λ between two successive levels ϵ_i and ϵ_{i+1}. It is reasonable to assume that it is mainly the fluctuations of these levels and their associated wavefunction amplitudes $\varphi_i(0)$ and $\varphi_{i+1}(0)$ which determine the position of λ. Second, if we are interested in the center of the resonance, i.e. states such that the distance $\lambda - \epsilon_0$ from the resonant level is significantly smaller than the width of the resonance (which is approximately $T_{\rm K}$), the right hand side of Eq. (21) can be neglected. Denoting $x_i = \mathcal{A}|\varphi_i(0)|^2$, we therefore get

$$\frac{x_\alpha}{\lambda - \epsilon_\alpha} + \frac{x_{\alpha+1}}{\lambda - \epsilon_{\alpha+1}} = 0 \; , \tag{23}$$

with the x_i's fluctuating according to (20). Interestingly, all energy scales beyond the spacing $(\epsilon_{\alpha+1} - \epsilon_\alpha)$ have disappeared from this equation: this toy model predicts a universal distribution of the ratio $(\lambda - \epsilon_\alpha)/(\epsilon_{\alpha+1} - \epsilon_\alpha)$. A straightforwad calculation gives

$$P^{\rm GOE}(\lambda) = \frac{1}{\pi \delta\epsilon} \frac{1}{\sqrt{(\epsilon_{\alpha+1} - \lambda)(\lambda - \epsilon_\alpha)}} \quad \text{and} \quad P^{\rm GUE}(\lambda) = \frac{1}{\delta\epsilon} \tag{24}$$

for the time reversal symmetric case (GOE, $\beta = 1$) and non-symmetric case (GUE, $\beta = 2$), respectively.

As a consequence we see that the correlations between the unperturbed energies ϵ and the perturbed ones λ come from two different mechanisms. On the one hand, *on energy scales larger than* Δ the constraints imposed by Eq. (21), in particular the fact that there can be only one state λ between two successive ϵ's, imposes a very strong correlation between the two sets of levels. On the other hand *on energy scales smaller than* Δ, these correlations depend a lot on the statistics of the wavefunction. Within our toy-model approach, there is no correlation in the GUE case, while for the GOE the λ's tend to cluster (with a square root singularity) near the ϵ's. Numerical simulations with the full resonant model confirm these latter predictions [7].

6. Conclusion

To conclude this short review, we have seen that studying the Kondo problem in the context of quantum dots makes it natural to consider how mesoscopic fluctuations affect strong correlations. The high temperature regime $T > T_K^0$ is the simplest to comprehend: reasoning based on the one-loop renormalization group approach makes it possible to incorporate all the effects of the mesoscopic fluctuations into those of the Kondo temperature itself, keeping otherwise all quantities universal, with the same universal functions as for the flat-band bulk. Universality is lost at temperatures below $T_{\rm K}$. However, the very low temperature regime $T, \Delta \ll T_{\rm K}^0$ can be tackled in the spirit of Nozières' Fermi liquid approach. As a first step, we have developed a slave boson/fermion mean field theory of this regime and obtained in this way a convincing qualitative description. The crossover between these two understood limiting cases, T of order $T_{\rm K}^0$, is, as usual, the most difficult part of the problem, and is still completely open.

Acknowledgments

The work at Duke University was supported in part by U.S. DOE award # DE-SC0005237. SB acknowledges support from french ANR projects SINUS and IsoTop.

References

[1] A.C. Hewson, *The Kondo Problem to Heavy Fermions* (Cambridge University Press, Cambridge, 1993).
[2] Éric Akkermans and Gilles Montambaux, *Mesoscopic Physics of Electrons and Photons* (Cambdrige University Press, Cambdrige, 2007).
[3] R. K. Kaul, D. Ullmo, S. Chandrasekharan and H. U. Baranger, *Europhys. Lett.* **71**, 6, 973 (2005).
[4] J. Yoo, S. Chandrasekharan, R. K. Kaul, D. Ullmo and H. U. Baranger, *Phys. Rev. B* **71**, 201309(R) (2005).
[5] R.K. Kaul, G. Zaránd, S. Chandrasekharan, D. Ullmo and H.U. Baranger, *Phys. Rev. Lett.* **96**, 176802 (2006).
[6] Ribhu K. Kaul, Denis Ullmo, Gergely Zaránd, Shailesh Chandrasekharan and Harold U. Baranger, *Phys. Rev. B* **80**, 3, 035318 (2009).
[7] Dong E. Liu, Sébastien Burdin, Harold U. Baranger and Denis Ullmo (2011). In preparation.
[8] M. Fowler and A. Zawadowzki, *Solid State Communications* **9**, 471 (1971).
[9] P. W. Anderson, *J. Phys. C: Solid State Physics* **3**, 12, 2436 (1970).
[10] A. A. Abrikosov, *Physics* **2**, 5 (1965).
[11] Kenneth G. Wilson, *Rev. Mod. Phys.* **47**, 4, 773 (1975).
[12] N. Andrei, *Phys. Rev. Lett.* **45**, 5, 379 (1980).
[13] P. B. Wiegmann, *Sov. Phys. JETP Lett.* **31**, 392 (1980).
[14] P. Nozières, *J. Low Temp. Phys.* **17**, 31 (1974).
[15] L.I. Glazman and M.E. Raikh, *Pis'ma Zh. Eksp. Teor. Fiz.* **47**, 378 (1988). [JETP Lett. **47** (1988) 452].
[16] Tai Kai Ng and Patrick A. Lee, *Phys. Rev. Lett.* **61**, 15, 1768 (1988).
[17] D. Goldhaber-Gordon, H. Shtrikman, D. Mahalu, D. Abusch-Magder, U. Meirav and M.A. Kastner, *Nature* **391**, 156 (1998).
[18] M. Grobis, I. G. Rau, R. M. Potok and D. Goldhaber-Gordon, in: *Handbook of Magnetism and Advanced Magnetic Materials, Vol. 5*, eds. H. Kronmüller and S. Parkin (Wiley, 2007), pp. 2703–2724. ArXiv:cond-mat/0611480.
[19] A. M. Chang and J. C. Chen, *Rep. Prog. Phys.* **72**, 096501 (2009).
[20] G. Zaránd and L. Udvardi, *Phys. Rev. B* **54**, 11, 7606 (1996).
[21] A. Crépieux and C. Lacroix, *Phys. Rev. B* **61**, 10, 6785 (2000).
[22] Stefan Kettemann and Eduardo R. Mucciolo, *Phys. Rev. B* **75**, 18, 184407 (2007).
[23] A. Zhuravlev, I. Zharekeshev, E. Gorelov, A. I. Lichtenstein, E. R. Mucciolo and S. Kettemann, *Phys. Rev. Lett.* **99**, 24, 247202 (2007).
[24] Denis Ullmo, *Rep. Prog. Phys.* **71**, 026001 (2008).
[25] Rainer Bedrich, Sébastien Burdin and Martina Hentschel, *Phys. Rev. B* **81**, 17, 174406 (2010).
[26] O. Bohigas, in: *Chaos and Quantum Physics*, eds. M. J. Giannoni, A. Voros and J. Jinn-Justin (North-Holland, Amsterdam, 1991), pp. 87–199.
[27] L. P. Kouwenhoven, C. M. Marcus, P. L. McEuen, S. Tarucha, R. M. Wetervelt and N. S. Wingreen, in: *Mesoscopic Electron Transport*, eds. L. L. Sohn, G. Schön and L. P. Kouwenhoven (Kluwer, Dordrecht, 1997), pp. 105–214.
[28] Madan Lal Mehta, *Random Matrices (Second Edition)* (Academic Press, London, 1991).